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FAMILY PLANNING, GROWTH, INCOME DISTRIBUTION: GRAPH-THEORETIC PATH ANALYSIS OF RWANDA

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This paper explores macroeconomic linkages among family planning, human capital and economic growth in Rwanda. Based on a disaggregated social accounting matrix (SAM), welfare effects of alternative exogenous injections are investigated, and the high and low-income pathways are identified by graph-theoretic path analysis. Three important findings follow from the analysis. First, rural income gains spread over the entire economy, while urban income gains are largely contained within urban areas. This suggests a relatively larger income multiplier effect of rural investments. Second, investing in family planning and health promotes agricultural production, with a considerable rural employment effect. Thus, targeted rural investment should yield economic growth followed by an improvement in income equality. Third, a unit increase in the consumption of family planning and health commodities is respectively associated with 1.3 unit, 1.2 unit and 0.74 unit increase in the agricultural, service and manufacturing production; It further generates 60% more income for the urban-Kigali households than rural households. To sum up, benefits of investing in family planning-health should not be overlooked in terms of improvement in rural employment, agricultural production and poverty reduction.

Keywords: Family Planning, Fertility, Human Capital, Growth, Income Distribution, Graph-Theory, Path Analysis, SAM Multiplier, Rwanda
JEL classification: I15, I25, I38, J13, O15, O21

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1. BACKGROUND AND INTRODUCTION

Family planning has long been a central component of population policies and programs and is an integral part of reproductive health.[1] It provides couples with methods of preventing unplanned pregnancies not only to reduce fertility and child mortality but also augment investment in child health. Since the 1960s, the use of family planning has been steadily increasing in the developing world. For Europe and the United States, for example, it took a century to reduce their average family size from around 6 to 3 children. For the developing world, however, a comparable decline in family size took only about four decades during 1960-2000. Despite the convergence in family size across the two worlds, unmet need for family planning in the developing world still remains about one-fifth of the currently married women.

Rwanda is no exception to high unmet need for family planning. Relative to other African countries, it is leading with its low contraceptive prevalence (36% in 2008), high fertility rate (5 children in 2009) and high unmet need for family planning (32% in 2008).[2] Data obtained from the 2008 DHS paint a picture of a rather unstable pattern of contraceptive use among married women. The contraceptive use sharply fell after the 1994 genocide, from 21.2% in 1992 to only 13.2% in 2000 and thereafter it slowly increased until 2005 to the point where 17.4% of married women were using modern contraception. By 2008, the contraceptive use has leveled around 36% - which can largely be attributed to the recent surge of investment in family planning services and increasing flow of donor funds targeting population programs.[3] At present, the main concern is the adverse effect of high unmet need and high fertility on per child human capital investment (i.e., education, health and nutrition) and economic growth through declining productivity.

Recognizing the link between fertility and development outcomes, the Rwandan government views family planning as an important instrument for targeting poverty and raising per child resource allocation at the household level. However, poverty is multi-sectoral in its cause, and decreasing family size can reduce it only partially. The creation of new employment opportunities is necessary for households to benefit from their investment in child quality because employment, sectoral productivity and household family size decision are interlinked at the meso level through economic and

[1] Reproductive health services include family planning, maternal health, childbirth, infant care and other personal reproductive health services for women. Health interventions relating to these services include contraceptive use, maternal health, infant and child health, neonatal and maternal morbidity and mortality, infertility technologies among others. Family planning would lower fertility through reducing obstacles to contraceptive use and access to reproductive health services.

[2] The author's own collection of data from the World Bank, UNDP, IMF and WHO online databases.

[3] See Solo (2008) for a comprehensive review of the developments in family planning in Rwanda over the period 1992-2008.

demographic policies.

In the literature, analysis of the economic effects of fertility usually focuses on an assessment of the rate of return to investment in human capital because high fertility puts mothers at risk, rises the dependency ratio and lowers per child investment in human capital, which in turn reduces productivity and income. A large number of micro-econometric and demographic studies show that family planning is the most direct and effective way to reduce fertility, making other interventions more effective in improving overall welfare, and that family planning is negatively associated with children's educational and health attainment (for example, Angrist, Lavy and Schlosser, 2005; Schultz, 2005; Rosenzweig and Zhang, 2009). Many studies also suggest that providing family planning services is the most direct and effective way to reduce fertility, making other interventions more effective in improving overall welfare (for example, World Bank, 1990; Ross, Parker, Green and Cooke, 1992; Schultz, 1997).

A recently growing literature focuses on macroeconomic analyses which integrate household fertility behavior with the consumption/saving decision. The models presented by Becker and Barro (1988) and Barro and Becker (1989), for example, demonstrate that fertility is inversely related to growth. At low levels of education, a combination of low productivity and high fertility point to a Malthusian equilibrium. With a general equilibrium model, Becker, Murphy and Tamura (1990) derives the conditions under which a country may switch from the Malthusian to the development equilibrium in which high levels of human capital stock lead to high productivity and low fertility. Their analysis highlights that a country may reach a reasonably high development level if it has good policies that favor human capital investment. More recently, the focus switched towards models that discuss demographic transition. For example, Galor and Weil (1996, 2000) argue that as a result of increased technological progress, the returns to education increases, causing a quality–quantity trade-off and hence, a fertility transition. Azarnert (2006) analyzes the impact of decline in child mortality on fertility and economic growth and shows that the timing of mortality relative to education is crucial to the implications of mortality decline. He also demonstrates the causal link between rising parental education and declining child mortality. The list can be extended at will.

Very little has been done about the analysis of macroeconomic effects of family planning within social accounting matrix (SAM) framework, although such analysis may provide critical information on effective targeting of specific household groups. Only a few studies have been carried out so far.[4] For example, Defourny and Thorbecke (1984) characterize the interactions among production, factors of production and households in

[4] On the contrary, there is a large number of studies applying the SAM multiplier method to analyze: growth strategies in developing economies (Pyatt and Round, 1985), technology and income distribution and (Roland-Holst and Sancho, 1992; James and Khan, 1997), fiscal policies (Whalley and Hillaire, 1987), intersectoral linkages and poverty (Thorbecke, 1995) among many others.

the context of South Korea. They demonstrate that when production activities are poorly linked, households facilitate the transmission of economic influence across production activities. Likewise, Roberts (1996) finds out that households play an important role in the establishment and strengthening of structural linkages between agriculture and the rest of the economy as well as in the rural-urban spillover. Examining the role of different household groups in the transmission of exogenous shocks within rural economies, Roberts (2005) further demonstrates that households with children are the most important transmitters of economic influence within the local economy examined and that large differences exist with respect to the dependence of different sectors on particular types of households. Another original study follows from Osorio, Carlos and Quentine (2010), adopting the SAM framework, explores the transmission channels through which sectorial growth patterns of Tanzania imply different effects on the incomes of women and men.

The purpose of this paper is to analyze macroeconomic linkages among family planning, sectoral growth and income distribution in Rwanda.[5] It is argued that increasing application of family planning activities is necessary to create savings and promote investment at the household level for education and health of children. Based on a disaggregated Social Accounting Matrix (SAM), macroeconomic effects of alternative income policies are assessed. The so-called structural path analysis of Defourny and Thorbecke (1984) has been commonly applied in the literature, aiming to identify critical pathways of average expenditure propensities behind SAM multipliers. To our knowledge, the literature lacks research applying the graph-theoretic path analysis (GPA) to explore the role that family planning and reproductive health play in the transmission of economic influences in the Rwandan economy. The current paper therefore applies the GPA to identify critical pathways of income multipliers with a view to accounting for the contribution of an exogenous injection to sectoral incomes (Cormen, Leiserson and Rivest, 1990; Hudson, 1992; Richardson, 1999). The GPA further helps identify the high and low-income pathways within the Rwandan economy.

Three important findings seem to evolve from the analysis. First, rural income gains spread over the entire economy, while urban income gains are largely contained within urban areas. This suggests a relatively larger income multiplier effect of rural investments. Second, investing in family planning and health promotes agricultural production, with a considerable rural employment effect. Targeted rural investment thus seems to bring growth and harmoniously improve income distribution. Third, a unit increase in the consumption of family planning and health commodities is respectively associated with 1.3 unit, 1.2 unit and 0.74 unit increase in the agricultural, service and manufacturing production; It further generates 60% more income for the urban-Kigali households than rural households. All in all, investing in family planning-health is a

[5] See Rosenzweig (1988) and Bloom, Canning and Sevilla (2001) for a through analysis of the linkages between population pressure and economic development.

viable strategy to create rural employment, increase agricultural production and reduce poverty.

The rest of the paper is organized as follows. The following section highlights the critical socioeconomic developments in Rwanda, with an examination of historical trends in unmet need and contraceptive use, fertility, child mortality and growth. Section 3 presents the SAM multiplier and the GPA. Section 4 describes available data and the adjustment of the existing SAM to incorporate family planning into the analysis. Section 5 discusses the key findings and their policy implications. Section 6 concludes the paper.

2. THE CRITICAL SOCIOECONOMIC DEVELOPMENTS

As seen from Figure 1, family planning and fertility in Rwanda have gone through three distinct periods. During the first period (1983-1994), a significant progress has been made in contraceptive use and a corresponding decline in fertility from about 8 to 6. However, unmet need for family planning has remained stable at 37%. During the second period (1994-2004), a mix picture emerged, with contraceptive use first declining and then stabilizing around 15%. Furthermore, fertility rate stabilized around 5.6 and unmet need remained unchanged. Finally, during the last period (2004-2009), all of the three indicators recorded significant changes. Contraceptive use jumped up more than 100% from 17% in 2005 to 36% in 2008; fertility rate dropped from 5.6 to almost 5; and unmet need dropped from 38% to 32%, which is a very significant decline for the first time since 1983. Two key factors responsible for the progress recorded since 2005 include large donor funding and government dedication and support for family planning activities (Solo, 2008). The role of the economic and political stability prevailing since early 2000 cannot be overlooked in the production of children as well as in the investment in education, health and nutrition of the existing children in a household.

Until 2008, many health indicators returned to pre-genocide levels. Mortality and fertility rates have also returned to pre-genocide levels in 1998 and thereafter continued to show a stable decline (see Figure 2). Infant mortality declined from 86 to 62; under-five mortality, from 152 to 103 (see Figure 3). Family planning programs have certainly played an important role in mortality and fertility decline through better birth spacing promoted by these programs, which not only reduces fertility but also improves maternal and child health which in turn reduces infant, under-five and maternal mortality.

Substantial evidence in the literature reveals that poverty cannot be reduced under high rates of population growth and that lowering fertility - in part through family planning - is essential. The 2008-2012 Economic Development and Poverty Reduction Strategy Paper of Rwanda (MINECOFIN, 2007) has for the first time acknowledged this link between poverty and fertility, stressing the importance of family planning not only as a health but also an economic intervention. Figure 4 shows that, after 2000, a negative relationship is observed between GDP per capita and fertility, which can in part be attributed to the double-intervention role of family planning. An increasing GDP per capita has in fact put forward a window of opportunity for Rwanda's young population, providing necessary resources for the needed human capital investment in education, health, and nutrition of children.

The sectoral distribution of value-added and employment figures reveals that Rwanda is approaching to a high economic growth trajectory. As seen from Figure 5, from 2000 on, sectoral growth has been undergoing a structural change. Agricultural value-added has declined; manufacturing has stabilized; and service value-added significantly grown. In line with these, agricultural employment declined from 85 % in 2001/02 to 71% in 2006/06, while off-farm self-employment increased (MINECOFIN, 2007). However, extreme land fragmentation and weak capital market impede the transition process to sustainable growth.

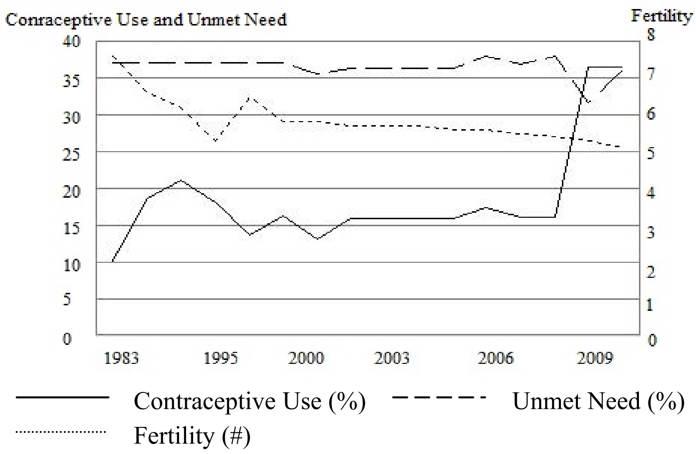

Figure 1. Contraceptive Use, Unmet Need and Fertility

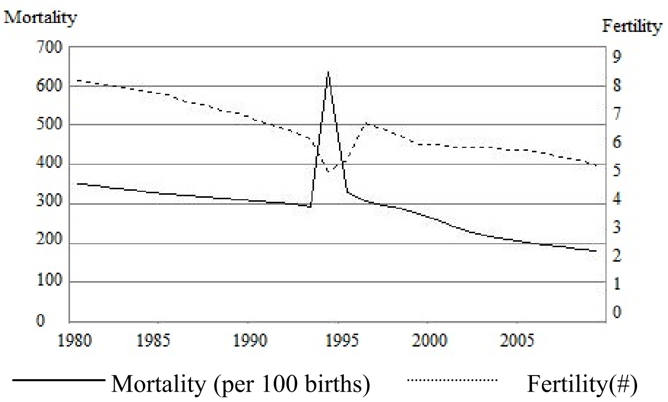

Figure 2. Mortality versus Fertility

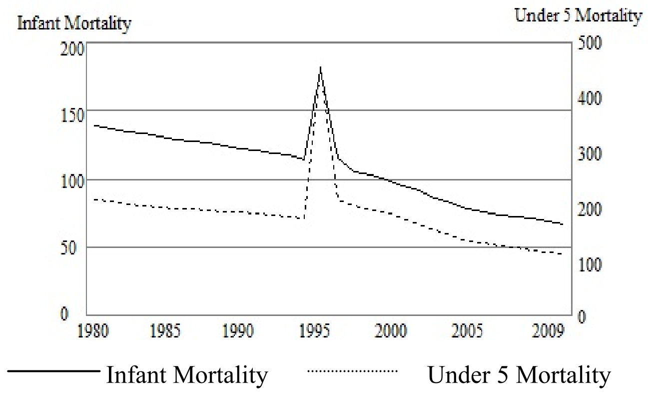

Figure 3. Infant versus Under 5 Mortality

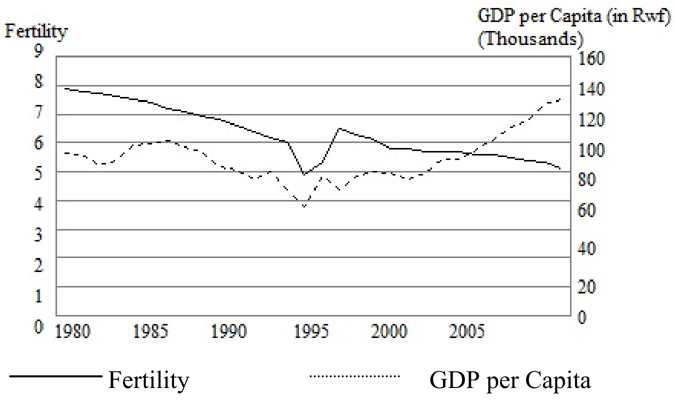

Figure 4. GDP per Capita and Fertility

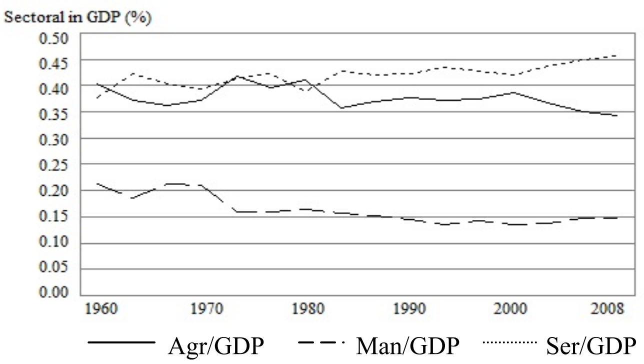

Figure 5. Sectoral Distribution of Value Added

3. METHODOLOGY

3.1. Accounting Multipliers

SAM is a matrix representation of the system of national accounts, where column sums (i.e., expenditures) are equal to row sums (i.e., incomes). To analyze a policy change, some accounts in the SAM must be manipulable exogenously; therefore, in a modeling framework, the SAM is partitioned as endogenous and exogenous accounts. Production activities, commodities, factors, households and firms represent endogenous accounts, while the government, savings-investment and the rest of the world accounts are assumed to be exogenous.

Let $T_{(d,d)} = [t_{ij}]_{i=j=1,...,d}$ denote a SAM with $d = (n + x)$ where n and x denote the number of endogenous and exogenous accounts, respectively. An element, t_{ij}, denotes the flow of money (real output) from account j (account i) to account i (account j). Let $T_{(d,d)}$ be partitioned as:

$$T_{(d,d)} = \begin{bmatrix} T_{nn} & T_{nx} \\ T_{xn} & T_{xx} \end{bmatrix}, \tag{1}$$

where T_{nn} = transactions among endogenous accounts,
T_{nx} = injections from exogenous into endogenous accounts,
T_{xn} = leakages from endogenous into exogenous accounts,
T_{xx} = residuals arising from interactions among exogenous accounts,
(N, X, L, R) = vectors of row sums of $(T_{nn}, T_{nx}, T_{xn}, T_{xx})$, respectively,
$y = (y_1,..., y_d) \equiv ((y_n),(y_x))$ = vector of row sums of $T_{(d,d)}$,
$y' = (y'_1,..., y'_d) \equiv ((y'_n),(y'_x))$ = vector of column sums of $T_{(d,d)}$.

Let $A_{(d,d)} = [a_{ij}]_{i=j=1,...d}$ denote a matrix of average expenditure propensities where $a_{ij} = (t_{ij}/y'_j)$ and $\sum_{i=1}^{d} a_{ij} = \sum_{i=1}^{d} (t_{ij} / y'_j) = 1$ for $\forall j = 1,2,..., d$. Let $A_{(d,d)}$ be partitioned as:

$$A_{(d,d)} = \begin{bmatrix} A_{nn} & A_{nx} \\ A_{xn} & A_{xx} \end{bmatrix}, \tag{2}$$

where A_{nn} is a square matrix of average expenditure propensities across n endogenous accounts; A_{xn} is a matrix of leakages; that is, the proportions of n

endogenous accounts that leak out as expenditure into x exogenous accounts; A_{nx} is a matrix of injections; that is, the proportions of expenditures of x exogenous accounts injected into n endogenous accounts; and A_{xx} is a matrix of residuals; that is, the proportions of expenditures circulated only among x exogenous accounts.
SAM accounting multiplier matrix’ M_{nn}, follows from:

$$\begin{aligned} y_n &= N + X = A_{nn} y_n + X \\ &= (I - A_{nn})^{-1} X = M_{nn} X, \end{aligned} \tag{3}$$

where $M_{nn} = (dy_n/dX) = (I - A_{nn})^{-1}$ measures the impact of unit change in aggregate demand, X, on the incomes of endogenous accounts, y_n.[6] There are two ways to conduct scenario analysis. The simplest and most commonly applied way is to deal with only one instrument (source: where an injection originates from) and one target (sink: where final effect takes place). Eq. (3) represents the model used for the analysis of a single, aggregate injection, whereas Eq. (4) below is used to analyze scenarios with multiple instruments and multiple targets. Replacing X in Eq. (3) with T_{nx} allows us to disentangle the individual effects of multiple injections from exogenous accounts in T_{nx}:

$$y_{nx} = M_{nn} T_{nx}, \tag{4}$$

where y_{nx} is a matrix of n rows and x columns. Each column in y_{nx} corresponds to a vector of endogenous incomes associated with a single exogenous account in T_{nx} such as the government.

3.2. Graph-Theoretic Path Analysis

The Rwandan economy is characterized using shortest and longest paths in a directed-graph M' (which denotes the transpose of global multiplier matrix M).[7]

[6] See Defourny and Thorbecke (1984) for the implication of unitary income elasticity and for the linkages between accounting and fixed-price multipliers. The lack of data on expenditure (income) elasticity does not allow us to compute marginal expenditure propensities associated with the SAM of Rwanda.

[7] For notational convenience, we drop the subscript n from M_{nn}. It should also be noted that path analysis is conducted using the transpose of M, denoted by M', only because the interpretation of multiplier effects associated with complicated pathways becomes easy to understand. For example, multiplier effect of a pathway $(i \to k \to v \to s \to j)$ can be easily denoted by $m_{ikvsj} = (m_{ik} * m_{kv} * m_{vs} * m_{sj})$ for all $m_{ij} \in M'$.

Following Cormen, Leiserson and Rivest (1990), we apply the *Dijkstra shortest path algorithm* to identify shortest paths (s) in M'. For the identification of longest paths (l), we apply the *Dijkstra algorithm* to M'_A whose elements are:

$$[m_{ij}^A]_{i,j,=1,2,...,n} = \begin{bmatrix} m_{ij}^A = \dfrac{\overline{m}}{m_{ij}}, \text{ if } m_{ij} \neq 0 \\ m_{ij}^A = 0, \quad \text{if } m_{ij} = 0 \end{bmatrix}_{i,j=1,2,...,n},$$

with $\overline{m} \geq m_{ij}$ where $\overline{m} = \text{argmax}\{m_{ij} \mid m_{ij} \in M'\}$. The construction of M'_A converts the small numbers in M' into large numbers and large numbers into small numbers. Therefore, the application of the *Dijkstra shortest path algorithm* to M'_A will yield the longest pathways. The original multiplier matrix M' will be used to calculate the longest pathway multipliers based on the longest pathways detected.

Two kinds of paths are relevant for our analysis: binary paths and multiple-account pathways. The set of binary paths is defined by $\{p_{ij}^z \in P^z \mid P^z =$ an n by n matrix of binary $type-z$ paths; $p_{ij}^z \equiv (i \to j)_z =$ a binary $type-z$ path from account i to account j where $i = j = 1,2,...,n$ and $z = s,l\}$. The set of multiple-account pathways is defined by $\{p_{ik...vj}^z \in P^z \mid P^z =$ an n by n matrix of multiple-account $type-z$ pathways; $p_{ik...vj}^z \equiv (i \to k \to ... \to v \to j)_z =$ a multiple-account $type-z$ pathway from i to j through the intermediate accounts $k,...,v$ where $i = k = ... = v = j$ $= 1,2,...,n$ and $z = s,l\}$. Economic influence multipliers associated with the binary and multiple-account paths above are defined by $\{e_{ij}^z \in E^z \mid E^z =$ an n by n matrix of binary $type-z$ influence multipliers; $e_{ij}^z \equiv m_{ij}^z =$ a binary $type-z$ influence multiplier associated with $p_{ij}^z \equiv (i \to j)_z$ where $i = j = 1,2,...,n$ and $z = s,l\}$. The set of multiple-account economic influence multipliers is then defined by $\{e_{ik...vj}^z \in E^z \mid E^z =$ an n by n matrix of multiple-account $type-z$ influence multipliers; $e_{ik...vj}^z = (m_{ik}^z * ... * m_{vj}^z) =$ a multiple-account $type-z$ influence multiplier associated with $p_{ik...vj}^z \equiv (i \to k \to ... \to v \to j)_z$ where $i = k = ... = v = j = 1,2,...,n$ and $z = s,l\}$. For a given (i, j), the resulting set of paths and their associated influence multipliers, $\{(p_{ij}^z, p_{ik...vj}^z) \in P^z, (e_{ij}^z, e_{ik...vj}^z) \in E^z$ where $z = s,l\}$, provides us with part of the information required for the characterization of the sectoral interactions in an economy.

Additional information about the structure of the economy concerned can be derived by applying the principles of the systems methodology. The principle of *"controllability"* of an economic system requires the identification of dominant, sub-ordinate and interactive accounts in M'.[8] We apply this principle to detect those sectors which are relevant from a policy intervention perspective. This principle calls for the revelation of cause-effect information embodied in M'.[9] Account i is said to cause account j if i purchases goods or services from j, and that account i is said to be affected (or influenced) by account j if j purchases goods and services from i. This implies that the sum of the elements in row i and column i of M' would, respectively, represent the degree of "Cause" (C_i) and "Effect" (Ef_i). A coordinate (C_i, Ef_i) obtained from M' would show the location of account i in a two-dimensional graph, measuring the extent to which account i causes the system and to which account i is influenced by the rest of the system. Based on this coordinate system, account i is classified as dominant if $C_i > Ef_i$, sub-ordinate if $C_i < Ef_i$ and interactive if $C_i = Ef_i$. Such grouping of endogenous accounts in a SAM would provide policy makers with critical information about: (1) dominant accounts which act as the "source" of an exogenous injection of income, (2) subordinate accounts which act as the "sink" of the final impact of that injection, and (3) interactive accounts which act as the intermediary poles for "transmission" of both causes and effects. The (C_i^z, Ef_i^z) coordinates obtained from E^z where $z = s, l$ would, respectively, represent lower and upper bounds for each account: $C_i^s \le C_i \le C_i^l$ and $Ef_i^s \le Ef_i \le Ef_i^l$.

4. DATA DESCRIPTION AND INTEGRATION

So far, the Ministry of Health of Rwanda has compiled 5 consecutive National Health Accounts (NHA). Two important issues are noted concerning these accounts. First, since Demographic and Health Surveys do not collect data on household-out-of-pocket general health expenses (HOP-GHE), the NHA approximates HOP-GHE using insurance companies' and service providers' revenues from households. Second, the NHA is disaggregated to facilitate sub-analyses such as the analysis of reproductive health-family planning, malaria, HIV/AIDS, etc. The NHA's reproductive health-family planning (NHA-RHFP) sub-analysis organizes data only for the RHFP sub-account.

[8] The systems methodology has been widely applied in the analysis of agricultural, environmental, and cross-cutting issues. For example, Goldsworthy and de Vries (1994) explore the systems approach to assess opportunities in the developing country agriculture.

[9] Note that backward and forward (BF) linkage analysis also relies on the same type of measurement.

Using data from the NHA-RHFP sub-account, the original SAM developed by Emini (2007) is adjusted to create a separate account called *reproductive health and family planning account* (see Table A1 for the associated production and commodity accounts placed in the 23rd and the 32nd rows and columns). Since the general health expenditure (GHE) given in the original SAM includes the RHFP expenditure also, we first disentangle the RHFP from the GHE by assuming that general health and RHFP production and consumption activities are subject to similar technologies, employ similar labor, capital, and intermediate commodities and face a similar demand structure. Due to the lack of data, we are forced to make such a rather restrictive set of assumptions which apply to all the calculations in what follows.

To construct the extended SAM with two new accounts (i.e., P_{fp} = RHFP production account and C_{fp} = RHFP commodity account), we need to approximate four sets of values for the new transactions under the new accounts. First, household demand for general health and RHFP services is derived based on the assumption that the demand for general health and reproductive health is homogenous. This implies a uniform expenditure elasticity of income across general health and reproductive health production. In order to integrate household-specific out-of-pocket expenditure for RHFP (denoted by HOP-RHFP) into the original SAM, the following operations in Box 1 are carried out. In the extended SAM used in the analysis, twenty regions have been aggregated over ten regions based on the coordinates of the regions.

Box 1. Notations and Definitions

Notation	Definition	Description
η_r^{NHA}	$= (H_r^{NHA}/H_g^{NHA})$	η_r^{NHA} = proportion of HOP-RHFP in HOP-GHE H_r^{NHA} = HOP-RHFP from NHA H_g^{NHA} = HOP-GHE from NHA, including RHFP
H_r^{SAM}	$= \eta_r^{NHA}\ H_g^{SAM}$	H_r^{SAM} = HOP-RHFP from original SAM H_g^{SAM} = HOP-GHE from original SAM, including RHFP
$\hat{H}_g^{SAM}$	$= (1-\eta_r^{NHA})\ H_g^{SAM}$	$\hat{H}_g^{SAM}$ = net HOP-GHE from original SAM, excluding RHFP
$\Sigma_{i=1}^{20}\theta_i$	$=1$	θ_i = proportion of households in region i
$H_{r,i}^{SAM}$	$= \theta_i\ H_r^{SAM}$ for $i = 1,..,20$	$H_{r,i}^{SAM}$ = HOP-RHFP of households in region i
$\hat{H}_{g,i}^{SAM}$	$= \theta_i\ \hat{H}_g^{SAM}$ for $i = 1,..,20$	$\hat{H}_{g,i}^{SAM}$ = net HOP-GHE of households in region i

The calculated figures in Box 2 are placed in the cells corresponding to rows (31 and 32) and columns (3 through 12) in Table A1.

Box 2. The Parameters Calculated for the Variables in Box 1

$\eta_r^{NHA} = 0.024$	$\theta_3 = 0.02$	$\theta_{13} = 0.04$
H_g^{SAM} = 10101 million Rwf	$\theta_4 = 0.03$	$\theta_{14} = 0.001$
H_r^{SAM} = 242 mil Rwf	$\theta_5 = 0.02$	$\theta_{15} = 0.12$
$\hat{H}_g^{SAM}$ = 9859 mil Rwf	$\theta_6 = 0.001$	$\theta_{16} = 0.04$
$\hat{H}_{g,1}^{SAM}$ = 3146 mil Rwf, where 1 denotes urban-Kigali H_{uk}	$\theta_7 = 0.001$	$\theta_{17} = 0.001$
$H_{r,1}^{SAM}$ = 19 mil Rwf, where 1 denotes urban-Kigali H_{uk}	$\theta_8 = 0.001$	$\theta_{18} = 0.001$
$\hat{H}_{g,2}^{SAM}$ = 371 mil Rwf, where 2 denotes urban-South H_{us}	$\theta_9 = 0.21$	$\theta_{19} = 0.13$
$H_{r,2}^{SAM}$ = 12 mil Rwf, where 2 denotes urban-South H_{us}	$\theta_{10} = 0.04$	$\theta_{20} = 0.05$
$\theta_1 = 0.08$	$\theta_{11} = 0.004$	
$\theta_2 = 0.05$	$\theta_{12} = 0.16$	

Secondly, we calculate the values for the definitions in Box 3.

Box 3. Notations and Definitions

Notation	Definition	Description
π_r^{NHA}	$= (P_r^{NHA}/P_g^{NHA})$	π_r^{NHA} = proportion of RHFP in GHE from NHA P_r^{NHA} = RHFP revenues of health insurance companies listed in NHA P_g^{NHA} = GHE revenues of health insurance companies listed in NHA, including RHFP
L_r^{SAM}	$= \pi_r^{NHA} L_g^{SAM}$	L_r^{SAM} = cost of labor in RHFP production from SAM L_g^{SAM} = costs of labor in GHE from SAM
$\hat{L}_g^{SAM}$	$= (1 - \pi_r^{NHA}) L_g^{SAM}$	$\hat{L}_g^{SAM}$ = net cost of labor in GHE from SAM, excluding RHFP
C_r^{SAM}	$= \pi_r^{NHA} C_g^{SAM}$	C_r^{SAM} = cost of capital in RHFP production from SAM C_g^{SAM} = cost of capital in GHE in SAM
$\hat{C}_g^{SAM}$	$= (1 - \pi_r^{NHA}) C_g^{SAM}$	$\hat{C}_g^{SAM}$ = cost of capital in GHE from SAM, excluding RHFP

The calculated figures reported in Box 4 are placed in the cells corresponding to rows (1 and 2) and columns (22 and 23) in Table A1.

Box 4. The Parameters Calculated for the Variables in Box 3

$\pi_r^{NHA} = 0.062$	$C_g^{SAM} = 5959$ mil Rwf
$L_g^{SAM} = 14333$ mil Rwf	$C_r^{SAM} = 371$ mil Rwf
$L_r^{SAM} = 893$ mil Rwf	$\hat{C}_g^{SAM} = 5588$ mil Rwf
$\hat{L}_g^{SAM} = 13444$ mil Rwf	

Thirdly, the general health (P_{he}) and RHFP production activities (P_{fp}) use agricultural (C_a), manufacturing (C_m) and service (C_s) commodities as intermediate inputs. The operations given in Box 5 are performed to calculate six intermediate consumption values.

Box 5. Notations and Definitions

Notation	Definition	Description
$C_{a,r}^{SAM}$	$= \pi_r^{NHA}\, C_{a,g}^{SAM}$	$C_{a,r}^{SAM}$ = P_{fp}'s inter. cons of agr output
		$C_{a,g}^{SAM}$ = P_{he}'s inter. cons of agr output, incl P_{fp}'s cons
$\hat{C}_{a,g}^{SAM}$	$= (1-\pi_r^{NHA})\, C_{a,g}^{SAM}$	$\hat{C}_{a,g}^{SAM}$ = P_{he}'s inter. cons of agr output
$C_{m,r}^{SAM}$	$= \pi_r^{NHA}\, C_{m,g}^{SAM}$	$C_{m,r}^{SAM}$ = P_{fp}'s inter cons of manufacturing output
$\hat{C}_{m,g}^{SAM}$	$= (1-\pi_r^{NHA})\, C_{m,g}^{SAM}$	$\hat{C}_{m,g}^{SAM}$ = P_{he}'s inter cons of manufacturing output
		$C_{m,g}^{SAM}$ = P_{he}'s inter cons of manufacturing output, including P_{fp}'s cons
$C_{s,r}^{SAM}$	$= \pi_r^{NHA}\, C_{s,g}^{SAM}$	$C_{s,r}^{SAM} = P_{fp}$'s inter cons of service output
$\hat{C}_{s,g}^{SAM}$	$= (1-\pi_r^{NHA})\, C_{s,g}^{SAM}$	$C_{s,g}^{SAM} = P_{he}$'s inter cons of service output, including P_{fp}'s cons
		$\hat{C}_{s,g}^{SAM} = P_{he}$'s net inter cons of service output

The calculated figures given in Box 6 are then placed in the cells corresponding to rows (24, 27 and 28) and columns (22 and 23) in Table A1.

Box 6. The Parameters Calculated for the Variables in Box 5

$C_{a,r}^{SAM} = (0.062 * 1028) = 64$	$C_{m,r}^{SAM} = (0.062 * 2395) = 149$	$C_{s,r}^{SAM} = 329$
$C_{a,g}^{SAM} = 1028$	$C_{m,g}^{SAM} = 2395$	$\hat{C}_{s,g}^{SAM} = 4945$
$\hat{C}_{a,g}^{SAM} = (1028 - 64) = 964$	$\hat{C}_{m,g}^{SAM} = (2395 - 149) = 2246$	

Finally, the values in the cells corresponding to rows (22, 23 and 39) and columns (31 and 32) express that all the general health and family planning outputs are purchased by the commodity sectors, and that theses sectors pay taxes to the government. Again, the ratio π_r^{NHA} is used to disentangle C_{he} and C_{fp}.

For completeness of exposition, **Box 7** defines the accounts in the extended SAM.

Box 7. The SAM Account Names

F_L	Labor	P_{he}	General health services
F_C	Capital	P_{fp}	Reproductive health-family planning services
H_{uk}	Households in urban Kigali	C_a	Agricultural commodity
H_{us}	Households in urban-South	C_{ax}	Agr. commody (exports)
H_{uw}	Households in urban-West	C_{fm}	Fishery, forsts, mining commodity
H_{un}	Households in urban-North	C_m	Manufacturing commodity
H_{ue}	Households in urban-East	C_s	Services received
H_{rk}	Households in rural Kigali	C_{padm}	Public administration services received
H_{rs}	Households in rural-South	C_e	Education services received
H_{rw}	Households in rural-West	C_{he}	General health services received
H_{rn}	Households in rural-North	C_{fp}	Reproductive health-family planning services received
H_{re}	Households in rural-East	C_{tm}	Commodity trade margin
H_{tra}	Household transfer	X_s	Service exports
F_i	Firms	X_{ml}	Exports of mining

P_a	Agricultural production	X_m	Manufacturing exports
P_{ax}	Agricultural exports	G	Government
P_{fm}	Production of fishery, forests, mining	T_d	Direct taxes
P_m	Manufacturing production	T_i	Indirect taxes
P_s	Service provision	SI	Savings-Investment
P_{padm}	Public services	RoW	Rest of the World
P_e	Education		

5. KEY FINDINGS AND THEIR POLICY IMPLICATIONS

This section presents the key findings obtained from multiplier analysis, scenario analysis and graph-theoretic path analysis. The critical policy implications of these findings are also discussed.

5.1. Multiplier Analysis

Eq. (5) presents the global multiplier matrix M, given in Table A2, in the form of six blocks of endogenous accounts: factor block (F) with labor and capital accounts, household block (H) with 11 accounts, firm (Fi) with one account, production block (P) with 9 accounts, commodity block (C) with 10 accounts and export block (X) with 3 accounts. Box 7 given above provides the names of the extended SAM accounts used in our analysis that follows.

$$M = \begin{bmatrix} M_{2,2}^{FF} & M_{2,11}^{FH} & M_{2,1}^{FF_i} & M_{2,9}^{FP} & M_{2,10}^{FC} & M_{2,3}^{FX} \\ M_{11,2}^{HF} & M_{11,11}^{HH} & M_{11,1}^{HF_i} & M_{11,9}^{HP} & M_{11,10}^{HC} & M_{11,3}^{HX} \\ M_{1,2}^{F_iF} & M_{1,11}^{F_iH} & M_{1,1}^{F_iF_I} & M_{1,9}^{F_iP} & M_{1,10}^{F_iC} & M_{1,3}^{F_iX} \\ M_{9,2}^{PF} & M_{9,11}^{PH} & M_{9,1}^{PF_i} & M_{9,9}^{PP} & M_{9,10}^{PC} & M_{9,3}^{PX} \\ M_{10,2}^{CF} & M_{10,11}^{CH} & M_{10,1}^{CF_i} & M_{10,9}^{CP} & M_{10,10}^{CC} & M_{10,3}^{CX} \\ M_{3,2}^{XF} & M_{3,11}^{XH} & M_{3,1}^{XF_i} & M_{3,9}^{XP} & M_{3,10}^{XC} & M_{3,3}^{XX} \end{bmatrix} = \begin{bmatrix} 7 & 29 & 0 & 30 & 31 & 10 \\ 7 & 41 & 0 & 29 & 31 & 10 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 7 & 39 & 0 & 40 & 42 & 13 \\ 9 & 52 & 0 & 42 & 55 & 14 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}. \quad (5)$$

An element M_{s_i,s_j}^{ij} is a sub-matrix of income multipliers between accounts in block i and accounts in block j where s_i and s_j stand for the number of accounts within block i and block j, respectively· For example, $M_{10,9}^{CP}$ is a sub-matrix of income multipliers between 9 production (P) and 10 commodity (C) accounts. The sum of the individual multipliers in $M_{10,9}^{CP}$ is equal to 42, implying that one unit injection into the production block P yields an increase of 42 units in the income of commodity block C.

Analysis of income transfers within the household block $M_{11,11}^{HH}$ - First of all, it should be noted that the extended SAM used in the analysis does not have data on the direct income transfers taking place among households, and that the flow of income within the household block is taking place indirectly either through consumption or production or factor accounts. As seen from the sub-matrix $M_{11,11}^{HH}$ given in Table A2, all of its diagonal elements are greater than one, implying that a unit injection into a household group generates for the same group an income larger than the initial injection. With a diagonal entry of 1.53, households in urban-Kigali (H_{uk}) are internally the most integrated within the household block, followed by $H_{rw} = 1.48$, $H_{re} = 1.47$, $H_{rs} = 1.42$ and so on. However, H_{uk} is found to be least integrated with other household groups in $M_{11,11}^{HH}$, reflected by its relatively low column sum of 3.13. This observation suggests that any income transfer to H_{uk} is more likely to create the lowest impact on the incomes of households in other regions of the country. On the contrary, rural households appear to be the most integrated with other household groups, reflected by column sums of $H_{rn} = 3.85$, $H_{rw} = 3.80$, and $H_{rs} = 3.79$. Namely, an improvement in rural incomes is widely spread across regions. This confirms that public policies should aim to strengthen the backward and forward linkages between urban-Kigali and rural areas for regional income gains to effectively spread over the entire economy.

Analysis of intermediate consumption patterns within the production block $M_{9,9}^{PP}$ - The input-output multipliers in $M_{9,9}^{PP}$ imply three important patterns. First, agricultural production has the largest income multiplier (i.e., the sum of the elements in the 1^{st} row of $M_{9,9}^{PP} = 13$), followed by services (5^{th} row sum $= 12$) and manufacturing (4^{th} row sum $= 8$). That means, of one Rwf injection into the production block, agriculture would benefit the most with an income gain of 32 cents (i.e., $13/40 = 0.33$), followed by services with 29 cents (i.e., $12/40 = 0.3$) and by manufacturing with 19 cents (i.e., $8/40 = 0.2$). In other words, agricultural, service and manufacturing production all together account for 83% of total multiplier (40) in $M_{9,9}^{PP}$; that is, $33/40 = 0.83$. The rest 17% is distributed across public sectors, including public administration, education, general health and RHFP accounts. Second, the cause-effect coordinates of

$M_{9,9}^{PP}$ show that agricultural, service and manufacturing production accounts are sub-ordinate, while public sectors are dominant accounts of the Rwandan economy.[10] Third, agricultural production (P_a) is internally the most integrated, implied by a diagonal multiplier of 2.34, followed by services (P_s) with 2.16 and manufacturing (P_m) with 1.75. The education, health and RHFP accounts are minimally integrated into the economy, which justifies the need for investment in health and RHFP to boost labor quality especially in rural areas. This would promote agricultural production, which would in turn reduce poverty.

Analysis of effects of changing household demand on production $M_{9,11}^{PH}$ - The $(1,1)^{th}$ element of $M_{9,11}^{PH}$, for example, measures the income multiplier effect on agricultural production of a unit increase in the income of H_{uk}. The analysis suggests that agriculture (40%), service (31%) and manufacturing (22%) production accounts for 93% of one Rwf injection into the household block. The contribution of public sectors is about 7%. The estimations also show that, on average, one Rwf increase in the transferable funds H_{tra} would yield an additional 3.5 Rwf for the entire production system. The findings further underline the presence of a strong rural-urban divide. More specifically, production effects of a change in rural household income are much larger than the effects of an equivalent change in urban household income. On average, of one Rwf injection into the household block, rural households tend to spend higher proportions on education, general health and RHFP relative to urban households. To sum up, investment in rural areas promises substantial improvement not only agricultural and service production but also in health and education demand.

Analysis of effects of changing production on household income $M_{11,9}^{HP}$ - Of one Rwf injection into the production system, 86% is absorbed only by 5 household groups: 28% by H_{uk} (i.e., 8/29 where 8 is the sum of the multipliers associated with the H_{uk} and 29 is the sum of the individual multipliers in $M_{11,9}^{HP}$), 17% by H_{rw} and H_{re} (i.e., 5/29), 14% by H_{rs} (i.e., 4/29) and 10% by H_{rn} (i.e., 3/29). Together, the sub-matrices of $M_{11,9}^{HP}$ and $M_{9,11}^{PH}$ present a consistent picture in which increasing GDP benefits the urban-Kigali area most (urban areas outside Kigali is negligible), followed by households in the rural west and east regions. This observation suggests that income effects of increasing production are contained within three regions, pointing to the need for strengthening forward and backward linkages between these three regions

[10] The degree of sector i's economic importance can be approximated by the analysis of its backward and forward linkages. Sector i's backward linkage measures the effect of a change in its final demand on all other sectors of the economy (i.e., cause), whereas its forward linkage measures the effect on sector i of a change in the economy-wide final demand (i.e., effect).

and the rest of the economy.

Analysis of the linkages among production, employment, technological change and poverty – Two pathways deserve special attention for our purposes. The first pathway $\{H \to C \to P \to F \to H\}$ is a closed-loop pathway. An income injection into the household block induces demand for commodities, which triggers production, which in turn increases the derived demand for factors, and then, the factor income earned is distributed back to households. The second pathway $\{P \to F \to H \to C\}$ starts with an exogenous increase in production, which raises factor demand, and the factor income earned is distributed across households. In turn, household demand for commodities increase.

The first pathway $\{H \to C \to P \to F \to H\}$ is examined through four sub-matrices $\{M_{10,11}^{CH}, M_{9,10}^{PC}, M_{2,9}^{FP}, M_{11,2}^{HF}\}$. $M_{10,11}^{CH}$ reveals that one Rwf injection into the household block induces a relatively more increase in rural households' demand for agricultural commodities. This suggests that relatively higher employment and hence income will be created in rural areas if households experience an income increase. On the other hand, one Rwf increase in the demand for education, general health and RHFP stimulates agricultural growth the most, followed by the growth of services and manufacturing production, which is implied by $M_{9,10}^{PC}$. This highlights the importance of investment in education-health-RHFP to pave the way for economic growth. With respect to technological change, $M_{2,9}^{FP}$ shows that agricultural, service and manufacturing sectors operate under labor-saving technologies because one unit increase in production yields 0.44 unit increase in labor use as opposed to 0.56 unit increase in capital use. Increasing education-health-RHFP production, on the other hand, induces a neutral technological change as implied by an equivalent rise in labor and capital use. All in all, the entire chain of interactions mapped by the sub-matrices concerned draw the following picture: increasing household income promotes rural households' consumption of agricultural commodities; increasing household demand for the education, health and RHFP commodities again promotes agricultural production the most; and increasing demand for agricultural output favors labor-saving technological change. Given these interactions, one can conjecture that (i) increasing household income may have an immiserizing (improving) welfare effect for those households substantially endowed with labor (capital);[11] and (ii) investing in education, health and RHFP promises improvement in agricultural, service and manufacturing production.

The second pathway $\{P \to F \to H \to C\}$ is analyzed through three sub-matrices $\{M_{2,9}^{FP}, M_{11,2}^{HF}, M_{10,11}^{CH}\}$. (Note that $M_{2,9}^{FP}$ and $M_{10,11}^{CH}$ have already been discussed in

[11] See Khan and Thorbecke (1989) for the application of the multiplier method to quantify macroeconomic effects of technological change.

the previous paragraph.) As seen from $M_{11,2}^{HF}$, H_{uk} relies relatively more on the income from labor employment, whose labor (capital) income corresponds to 0.27% (24%) of the economy-wide labor (capital) income. This is a typical observation in a developing economy in which urbanized capital city attracts low income or unemployed people around itself as it offers marginal employment opportunities. On the contrary, households in rural areas as a whole rely relatively more on capital income, whose capital (labor) income corresponds to 18% (14%) of the economy-wide capital (labor) income. What this implies together with the conclusion in the previous paragraph is that urban (rural) households would be the ones that suffer (benefit) from labor-saving technological change the most. However, since this technological change would improve rural productivity, income loss due to labor saving technological change would be compensated by productivity increase. In an economy with an abundant amount of unskilled labor, labor-saving technological progress would be a economically and socially viable strategy if the labor released from unproductive activities are re-gained into the economy through good employment policies. This suggests that technological progress and employment policy should go hand in hand to ease the negative consequences of technological change. Finally, an urban-rural divide is observed when we look at the distribution of economy-wide factor income across urban and rural household groups. Those household groups in urban areas as a whole earn significantly more factor income than those in rural areas. This should be attributed to higher wages in urban areas and high rental rates in rural areas because we know that a very large share of the population earns income from rural, specifically agricultural, activities.[12]

To investigate closed-loop, open-loop and transfer income multiplier effects, we decompose M following Defourny and Thorbecke (1984).[13] The calculations show that the most important closed-loop interactions start from the commodity block and return back to itself, which is implied by a net multiplier effect of 23. The important open-loop interactions, however, take place between the household and production

[12] Our observation that "urban households rely more on labor income and rural households rely on capital income" is merely a reflection of wage and rental rate differences between urban and rural areas. The SAM is constructed by using current expenditures and incomes, and does not imply much about the real amount of capital nor the number of employed. We know from the data that rural sector feeds a very large number of families as opposed to urban sectors. However, higher wages in urban areas and higher rental rates in rural areas due to the scarcity of machinery yield the result concerned.

[13] A detail analysis of the decomposition of M will be available upon request. Define M as:

$$M = \underbrace{I}_{\text{Initial in j.}} + \underbrace{(M_1 - I)}_{\text{Transfer multiplier}} + \underbrace{(M_2 - I)M_1}_{\text{Open-loop multiplier}} + \underbrace{(M_3 - I)M_2 M_1}_{\text{Closed-loop multiplier}},$$

where A_0 = block diagonal matrix constructed from diagonal blocks of A_n; $M_1 = (I - A_0)^{-1}$; $A^* = M_1(A_n - A_0)$; $M_2 = (I + A^* + A^{*^2} + \ldots + A^{*^{(n-1)}})$; and $M_3 = (I - A^{*^n})^{-1}$.

blocks, with a net multiplier of 18. Finally, the critical transfer interactions take place within both the production (16) and the household (16) blocks. These figures uncover a fundamental characteristic of the Rwandan economy. Exogenous income injections are welfare improving if they originate from the commodity and household blocks. Namely, policies aimed to boost demand is expected to lead welfare gains.

5.2. Scenario Analysis

Policy scenarios are analyzed using factual and counter-factual SAMs. The 2006 SAM represents the factual SAM since it reflects the already realized economic relations in Rwanda. The analysis based on the factual SAM therefore explores the implications of what has happened in 2006. A SAM with hypothetical exogenous injections, on the other hand, represents the counter-factual SAM since it indicates the unrealized but likely economic relations. The counter-factual SAM is used to assess "what if" scenarios and attempts to draw a picture of the future implications of the postulated economic relations.

Factual scenario (1) assumes 10% increase in the transfer of the RoW to measure the impact of Rwanda's exports and remittance revenues on different household groups. The raw data show that remittances from abroad play a critical role in households living in urban areas, especially Kigali region. The sum of the multipliers in $M_{11,11}^{HH}$, which is equal to 41, implies that, on average, an income transfer of 100 Rwf from the RoW into households is translated into an additional 4100 Rwf for households. According to this scenario, 10% increase, corresponding to an income transfer of 1192 mil Rwf, generates an additional income of 16551 mil Rwf for the entire economy. This national income gain is distributed across 4 accounts as follows: 5354 mil (33%) goes to commodity sectors; 4145 mil Rwf (25%), to households; 3982 mil (24%), to production sectors; and 2969 mil Rwf (18%), to factors. The income gain distributed points out that the RoW's direct income transfer (i.e., $RoW \rightarrow H$) has a second round indirect effect on household income through an increase in factor demand (i.e., $H \rightarrow C \rightarrow P \rightarrow F \rightarrow H$). Furthermore, the regional distribution of household income gain shows an income disparity between urban and rural areas. The transfer brings 1.84% increase in the income of urban households as opposed to 1.18% increase in the income of rural households, which reflects the fact that urban households receive a bigger proportion of the RoW's transfer. Lastly, we observe that the rise in the education, general health and RHFP commodity demand is not negligible at all.

Factual scenario (2) assumes 10% increase in the demand for manufacturing commodity. This scenario concerns the effects of raising manufacturing demand on the economy-wide income growth and the sectoral distribution of the income generated. Under this scenario, the economy-wide income increases by 3.1% of which 29% goes to households (14.2% to urban and 14.8% to rural households); 25%, to the commodity sectors; and 20%, to the production sectors. At the subsectoral level, in particular, the demand for agricultural export, manufacturing, services and domestic agricultural commodities rises substantially. Education, general health and RHFP commodity sectors receive 2.6% of the economy-wide income gain. The economic influence on the public sectors of the investment concerned takes place indirectly through interactions between households and commodity sectors.

Factual scenario (3) separately postulates 10% increase in the demand for education, health and RHFP commodities to characterize the economy-wide impetus in income generation. First, 10% increase in the education demand generates 1.2% increase in the economy-wide income. This additional income is equally distributed among households (13% of 1.2), production (12%) and commodity (13%) accounts. Secondly, when the demand for health increases, the economy-wide income raises 0.36%, 19% of which goes to households, 27% to production and 34% to commodity accounts. Thirdly, a 10% increase in the family planning demand yields 0.03% increase in the economy-wide income, which follows the same distribution pattern as in the health demand. These findings suggest that improved education has a neutral income effect on the rest of the economy, while improved health and family planning create favorable conditions for the commodity and production sectors through the positive impact of improved health on labor productivity.

Factual scenario (4) aims to analyze the sectoral and regional income effect of three simultaneous injections: (i) 5% increase in the investment in the manufacturing commodity sector, (ii) 5% increase in the provision of the RHFP services and (iii) 5% increase in the income of urban-Kigali households through the transfer from RoW. The total amount injected into the economy is 678 mil Rwf, which yields a net benefit of 127040 mil Rwf. Of this, 97% is attributed to the investment in manufacturing; 2% due to the RoW's transfers and 1% due to the provision of the RHFP services. The agricultural, manufacturing and service sectors each receives 25% of 127,040 mil Rwf; 11% goes to rural households; 9%, to factors of production; and 5%, to urban-Kigali households. When factual scenario (4) is run with an income transfer to rural rather than to urban-Kigali households, a higher level of national income is obtained. This justifies the implementation of policies that favor rural households and agricultural sector for poverty reduction. The findings from *a counter-factual scenario (1),* which contrasts the effects of 100 mil Rwf government subsidy across rural and urban households, further confirm that rural household subsidy is welfare improving over urban subsidy, with an economy-wide income gain of 1538 Rwf compared to a gain of 1371 Rwf.

Counter-factual scenario (2) aims to analyze whether equivalent simultaneous injections from three accounts into the economy have comparable implications for the economy-wide income growth. An injection of 157 mil Rwf is made through three channels: 55 mil Rwf from the RoW to H; 52 mil Rwf from the government to C_{fp}; and 51 mil Rwf from the S-I account to P_{fp}. *Counter-factual scenario (3)* assumes the same amount of injection but the allocations across the three channels are adjusted: 55 mil Rwf from the RoW to H; 72 mil Rwf from the government to C_{fp}; and 32 mil

Rwf from the S-I account to P_{fp}. The scenario (3) is welfare improving over the scenario (2), revealing that an increase in the public provision of family planning services promises significant income growth. Comparing the efficiency of the three sources of injection, we find that the government intervention yields the highest income gain (36% of the national income gain), followed by the intervention through the S-I account (34%) and lastly by the transfer from the RoW (30%). The observed effects in income growth are comparable, and therefore, the choice of policy instrument is not so important.

Counter-factual scenario (4) aims to analyze the impact of increasing investment in family planning services through an injection of 78 mil Rwf by the government and the S-I accounts. The consumption of family planning commodities is not influenced much in spite of a significant rise in the supply of these commodities, which clearly points to the fact that households are more likely to raise their demand for family planning if they realize the full benefits (monetary and non-monetary) of family planning programs. Thus, the provision of family planning products needs to be coupled with policies aimed to raise awareness about the full benefits that households can obtain from the participation in the family planning programs concerned.

To sum up, scenario analysis confirms the view that the agricultural sector and rural households should be targeted for poverty reduction, and that the SAM account through which an income injection is made is not critical for the generation of income gains. Furthermore, public awareness activities should target effective participation in family planning programs. Last but not least, the underlying parameters reflected by the income multiplier matrix imply that family planning and reproductive health does not occupy as much an important place as it should in the policy agenda of the government, although they have considerable income multiplier effects on the rest of the economy.

5.3. Path Analysis

Drawing on M', the path analysis identifies the effective pathways of economic influences between family planning, sectoral growth and income distribution. This would provide information on the "effective instrument(s)" and the "effective target(s)" of a policy intervention. Additional useful information is also obtained by identifying the dominant, sub-ordinate and interactive sectors in the Rwandan economy. Since our goal is to shed light on the interactions between family planning-general health, sectoral growth and income distribution, we limit our search for effective pathways of interactions between production, consumption and household accounts. The GPA addresses the following questions.

Which pathways from family planning-general health commodities to households (i.e., $C_{fp} \rightarrow H$ *and* $C_{he} \rightarrow H$ *) yield the maximum (minimum) income gain?* Table A3 shows that all the maximum-gain pathways from C_{fp} and C_{he} to households are direct without any intermediate sectors. Mapping the influence multipliers associated with these pathways, Table A4 shows that one unit exogenous increase in the demand for family planning and general health commodities generates 0.84 unit income gain only for H_{uk}. Income gain of rural households is significantly less than that of the urban-Kigali households, with 0.56 unit for H_{re}, 0.55 unit for H_{rw} and 0.48 unit for H_{rs}. Regarding the minimum-gain pathways, Table A5 shows that H_{ue}'s and P_{padm}'s demand for family planning commodities impede the income gain of households in urban-Kigali and urban-south regions**.**

Which pathways from households to family planning-general health consumption (i.e., $H \rightarrow C_{fp}$ and $H \rightarrow C_{he}$) yield the maximum (minimum) income gain? Table A3 indicates that the majority of the maximum-gain pathways, $H \rightarrow C_{fp}$ and $H \rightarrow C_{he}$, involve F_C and H_{rs} as intermediate poles which have large multiplier effects on the consumption of family planning and health services. One unit exogenous increase in household income generates the maximum demand for family planning and health commodities when this additional household income raises the demand for capital owned by households in the rural-south region or is directly transferred to the same households. On the contrary, Table A5 reveals that, of one unit increase in household income, a small portion is spent on family planning.

Does increasing demand for family planning-general health promote production (i.e., $C_{fp} \rightarrow P$ and $C_{he} \rightarrow P$)? The income multipliers in Table A4 show that one unit exogenous injection to C_{fp} and C_{he} commodity sectors results in 1.3 unit increase in the agricultural production, followed by 1.21 unit increase in the production of the service sector and 0.74 unit increase in the manufacturing production. This finding suggests that investing in RHFP and general health can also be viewed as an investment in productivity improvement in the key sectors of the Rwandan economy through improved labor productivity. An interesting observation is that an equal amount of income injection to the service and manufacturing commodity sectors leads to an income gain in the agricultural production much smaller than the gain implied by the RHFP and health commodity demand. Specifically, this confirms that investing in family planning has a higher return relative to the investment in the service and manufacturing sectors.

Which intermediate poles most (least) effectively transmit economic influence from the source to the sink? Table A3 reveals that labor, capital, households in H_{uw}, H_{rs}, H_{rn} and H_{re}, the manufacturing production and the agricultural consumption sectors would create significant income multiplier effects when any of them acts as an intermediate pole in between the source of income injection and the sink of the final impact of that injection. Two critical policy implications make themselves known: (1) the promotion of the manufacturing sector and agricultural development should go hand in hand to achieve economic growth and address poverty and (2) policies should target households in the urban-west, rural-south, rural-north and rural-east to stimulate not only family planning but also education and general health sectors. On the other hand, as seen from Table A5, households in H_{ue} and H_{tra}, C_{fp}, C_{padm}, and P_{padm} suppress the existing direct path multipliers, which in turn reduce the multiplier effects of the pathways they are involved in.

Finally, household transfers point to a strong urban-rural divide. Except for the households in urban-Kigali, rural-west, rural-east, rural-south and rural-north receive a much higher proportion of one unit income transfer compared to households in other urban regions.

Table 1. Cause-Effect Coordinates of M'

	Minimum Cause	Influence Effect	Optimal Cause	Influence Effect	Maximum Cause	Influence Effect
F_L	10	10	15	49	17	49
F_C	10	10	15	58	17	58
H_{uk}	14	6	12	28	16	28
H_{us}	10	4	14	6	18	8
H_{uw}	14	3	14	4	19	6
H_{un}	10	4	14	3	18	5
H_{ue}	14	3	14	3	18	5
H_{rk}	8	3	15	3	19	5
H_{rs}	7	6	15	17	18	19
H_{rw}	8	6	15	19	18	21
H_{rn}	7	5	16	13	19	16
H_{re}	8	5	15	19	18	21
H_{tra}	6	3	16	3	19	3
F_i	36	2	1	5	36	9
P_a	9	10	16	49	18	49
P_{ax}	7	3	16	3	19	5
P_{fm}	9	3	16	7	19	7
P_m	5	16	16	28	18	28
P_s	5	23	16	42	18	42
P_{padm}	5	2	16	2	19	3
P_e	7	3	16	5	19	6
P_{he}	6	2	16	3	19	4
P_{fp}	6	2	16	2	19	3
C_a	8	11	17	51	19	50

C_{ax}	6	3	17	2	20	4
C_{fm}	7	3	17	5	20	5
C_m	11	27	11	53	14	53
C_s	10	23	16	42	19	42
C_{padm}	5	2	17	1	20	2
C_e	7	3	17	4	20	6
C_{he}	7	2	17	2	20	3
C_{fp}	7	2	17	1	20	2
C_{tm}	10	6	17	11	20	16
X_a	8	36	17	1	19	36
X_{mi}	8	36	17	1	19	36
X_m	4	36	17	1	19	36

From the $C-Ef$ coordinates of M' presented in Table A1, households and public sector (including education, general health, RHFP and public administration) are found to be the *dominant* accounts; agriculture and service production and commodity sectors, to be the *interactive* accounts; and the manufacturing commodity sector, to be the *sub-ordinate* account. The dominant accounts represent the source of policy interventions, while the sub-ordinate accounts show the sinks of the final impact of these interventions. This means that the dominant accounts should be treated like exogenous factors in a modeling context. Our findings suggest that stimulating household and public sector demand is a viable option for policy makers to exert influence on other sectors of the economy and that the final impact should be first assessed in the manufacturing commodity sector and thereafter in other sectors linked to the manufacturing sector.

6. SUMMARY AND CONCLUSIONS

This paper explores macroeconomic linkages among family planning, human capital and growth in Rwanda. Based on a disaggregated SAM, welfare effects of alternative exogenous injections are examined, and the high and low-income pathways, identified by graph-theoretic path analysis.

Three important findings seem to evolve from the analysis. First, rural income gains spread over the entire economy, while urban income gains are largely contained within urban areas. This suggests a relatively larger income multiplier effect of rural investments. Second, investing in family planning and health promotes agricultural production, with a considerable rural employment effect. Targeted rural investment thus seems to bring growth and harmoniously improve income distribution. Third, a unit increase in the consumption of family planning and health commodities is respectively associated with 1.3 unit, 1.2 unit and 0.74 unit increase in the agricultural, service and manufacturing production; It further generates 60% more income for the urban-Kigali households than rural households. All in all, investing in family planning-health is a viable strategy to create rural employment, increase agricultural production and reduce poverty.

Some final remarks should be made on the limitation of the current study. First, the SAM data framework assumes that expenditure of an account represents the influence of that account on other accounts. In fact, the actual influence of one account on other accounts can be better approximated through a more detailed econometric causality estimation between the relevant accounts. Second, the multiplier analysis draws on average expenditure propensities obtained from the SAM, while marginal propensities are more reliable to depict non-linear structural relations. The implicit assumption of unitary expenditure elasticities may not show the actual behavior of an account and hence the SAM multiplier analysis employing the average propensities may not reflect the actual interactions among institutions.

Two critical issues warrant further research. The current paper postulates that general health and family planning production activities employ the same technologies and face with the same demand parameters. For a representative characterization of the family planning sector, an econometric time-series estimation should be carried out to estimate the technology and demand parameters concerned. Secondly, a SAM is a representation of aggregate economic identities, whereas family planning can only be characterized by a behavioral relation at the household level. The effects of family planning on economic indicators can be effectively analyzed by incorporating households and their family size decisions in a general equilibrium model and such a model certainly calls for the construction of a SAM including socioeconomic indicators.

APPENDIX

Table A1. Rwanda SAM 2006 (million Rwf)

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m	[19] P_s	[20] P_{padm}	[21] P_e
F_L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	262077	10021	25604	62785	19623	93331	44111
F_C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	325106	12439	30807	136884	248421	0	18339
H_{uk}	224466	147440	0	0	0	0	0	0	0	0	0	0	3330	0	0	0	0	0	0	0	0
H_{us}	36294	30863	0	0	0	0	0	0	0	0	0	0	1034	0	0	0	0	0	0	0	0
H_{uw}	19615	19247	0	0	0	0	0	0	0	0	0	0	377	0	0	0	0	0	0	0	0
H_{un}	14817	12687	0	0	0	0	0	0	0	0	0	0	358	0	0	0	0	0	0	0	0
H_{ue}	14761	13254	0	0	0	0	0	0	0	0	0	0	415	0	0	0	0	0	0	0	0
H_{rk}	10528	11666	0	0	0	0	0	0	0	0	0	0	350	0	0	0	0	0	0	0	0
H_{rs}	92401	122499	0	0	0	0	0	0	0	0	0	0	2389	0	0	0	0	0	0	0	0
H_{rw}	103357	139744	0	0	0	0	0	0	0	0	0	0	4181	0	0	0	0	0	0	0	0
H_{rn}	71590	91724	0	0	0	0	0	0	0	0	0	0	3344	0	0	0	0	0	0	0	0
H_{re}	106437	141670	0	0	0	0	0	0	0	0	0	0	3322	0	0	0	0	0	0	0	0
H_{tra}	0	0	4177	1611	642	398	525	298	2430	3481	2657	2880	0	0	0	0	0	0	0	0	0
F_i	0	47162	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{ax}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{fm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{padm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_e	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P_{he}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

P_{fp}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_a	0	0	52629	23915	9980	8849	6338	8541	109590	124375	90411	105163	0	0	15364	0	0	90136	7644	11541	5083
C_{ax}	0	0	2	0	0	1	0	0	32	44	35	67	0	0	0	0	0	27314	0	0	0
C_{fm}	0	0	8888	2035	936	1092	667	1480	6073	7177	5300	13125	0	0	0	0	2214	4148	8	0	0
C_m	0	0	139258	25018	15641	12167	12482	8710	70472	78808	47604	91173	0	0	16169	1196	4096	91602	58522	28397	6510
C_s	0	0	86195	12148	7534	4196	5388	2690	18428	22958	14315	26320	0	0	3325	3125	4982	110737	83157	36611	15171
C_{pdn}	0	0	505	55	56	36	30	10	68	139	89	245	0	0	0	0	0	0	0	0	0
C_e	0	0	13110	1034	913	345	1263	324	5213	4182	2461	5474	0	0	0	0	0	0	0	0	0
C_{he}	0	0	3146	371	298	215	136	159	1574	1535	668	1757	0	0	0	0	0	0	0	0	0
C_{fp}	0	0	19	12	4	7	4	2	59	49	39	47	0	0	0	0	0	0	0	0	0
C_{tm}	0	0	0	0	0	0	0	0	0	0	0	0	00	0	0	0	0	0	0	0	0
X_a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{mi}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Td	0	0	25355	1842	2486	331	759	49	1244	1340	721	1589		28820	0	0	0	0	0	0	0
Tl	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sl	0	0	44836	2927	944	893	923	288	2061	3261	2356	3789	0	38687	0	0	0	0	0	0	0
RoW	14620	0	1285	249	58	4	5	4	1362	727	537	844	0	19095	0	0	0	0	0	0	0
Total	708886	777955	379406	71216	39492	28532	28520	22556	218605	248076	167193	252475	19100	86602	62041	26781	67703	52606	59376	169890	89214

Table A1. (Continued)

	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{prdm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m	[37] G	[38] Td	[39] Tl	[40] Sl	[41] RoW	[1-41] Total
F_L	13440	893	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	708886
F_C	5588	371	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	777956
H_{uk}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4171	379406
H_{us}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3026	71216
H_{uw}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	254	39492
H_{un}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	669	28532
H_{ue}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	90	28520
H_{rk}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	22556
H_{rs}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1316	218605
H_{rw}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	795	248076
H_{rn}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	534	167193
H_{re}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1046	252475
H_{tra}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19100
F_i	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8576	0	0	0	30864	86602
P_a	0	0	620558	0	0	0	0	0	0	0	0	0	1483	0	0	0	0	0	0	0	622041
P_{ax}	0	0	0	26781	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26781
P_{fm}	0	0	0	0	49610	0	0	0	0	0	0	0	0	18093	0	0	0	0	0	0	67703
P_m	0	0	0	0	0	467511	0	0	0	0	0	0	0	0	15695	0	0	0	0	0	523606
P_s	0	0	0	0	0	0	594376	0	0	0	0	0	0	0	0	0	0	0	0	0	594376
P_{prdm}	0	0	0	0	0	0	0	169890	0	0	0	0	0	0	0	0	0	0	0	0	169890
P_e	0	0	0	0	0	0	0	0	89214	0	0	0	0	0	0	0	0	0	0	0	89214
P_{he}	0	0	0	0	0	0	0	0	0	27183	0	0	0	0	0	0	0	0	0	0	27183
P_{fp}	0	0	0	0	0	0	0	0	0	0	1806	0	0	0	0	0	0	0	0	0	1806
C_a	964	64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	670587
C_{ax}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	27495

C_{fm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	53143
C_m	2246	149	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	233514	0	943733
C_s	4945	329	0	0	0	0	0	0	0	0	0	150180	0	0	0	0	0	0	0	0	612745
C_{padm}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	168658	0	0	0	0	169890
C_e	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	54896	0	0	0	0	89214
C_{he}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17339	0	0	0	0	27198
C_{fp}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1565	0	0	0	0	1807
C_{tm}	0	0	48554	651	3533	97422	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15080
X_a	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1483	1483
X_{mi}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18093	18093
X_m	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	56095	56095
G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	64536	129109	0	168989	362635
Td	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	64536
Tl	0	0	100	5	0	110619	18369	0	0	16	1	0	0	0	0	0	0	0	0	0	129109
Sl	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2257	0	0	0	134805	233514
RoW	0	0	1375	58	0	268162	0	0	0	0	0	0	0	0	0	113858	0	0	0	0	422242
Total	27183	1806	670587	27495	53143	543733	612745	169890	89214	27199	1807	150180	1483	18093	56095	362635	64536	129109	233514	422242	

Table A2. Multiplier Matrix M

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m
F_L	2.098	1.088	0.940	1.128	1.071	1.121	1.087	1.194	1.237	1.237	1.260	1.201	1.168	0.000	1.518	1.490	1.489	1.320
F_C	1.405	2.393	1.202	1.449	1.375	1.444	1.391	1.534	1.580	1.583	1.611	1.538	1.494	0.000	1.927	1.894	1.879	1.804
H_{uk}	0.938	0.806	1.532	0.641	0.608	0.637	0.617	0.677	0.699	0.700	0.713	0.680	0.836	0.000	0.854	0.838	0.835	0.767
H_{us}	0.166	0.153	0.098	1.118	0.112	0.117	0.114	0.125	0.129	0.129	0.131	0.125	0.176	0.000	0.157	0.154	0.153	0.141
H_{uw}	0.094	0.090	0.057	0.068	1.065	0.068	0.065	0.072	0.074	0.074	0.076	0.072	0.090	0.000	0.091	0.089	0.089	0.082
H_{un}	0.068	0.063	0.040	0.048	0.046	1.048	0.046	0.051	0.052	0.053	0.054	0.051	0.068	0.000	0.064	0.063	0.063	0.058
H_{ue}	0.069	0.064	0.041	0.049	0.047	0.049	1.047	0.052	0.054	0.054	0.055	0.052	0.073	0.000	0.064	0.064	0.064	0.059
H_{rk}	0.053	0.053	0.033	0.039	0.037	0.039	0.038	1.042	0.043	0.043	0.044	0.042	0.059	0.000	0.052	0.051	0.051	0.047
H_{rs}	0.500	0.524	0.317	0.382	0.362	0.379	0.367	0.403	1.416	0.417	0.424	0.405	0.519	0.000	0.057	0.498	0.495	0.461
H_{rw}	0.568	0.598	0.631	0.437	0.414	0.433	0.419	0.460	0.474	1.476	0.485	0.462	0.668	0.000	0.577	0.567	0.564	0.525
H_{rn}	0.385	0.400	0.243	0.294	0.279	0.292	0.283	0.310	0.319	0.320	1.326	0.311	0.478	0.000	0.388	0.381	0.379	0.353
H_{re}	0.579	0.607	0.367	0.443	0.419	0.440	0.425	0.467	0.482	0.483	0.492	1.469	0.630	0.000	0.587	0.576	0.573	0.534
H_{tra}	0.045	0.044	0.038	0.056	0.048	0.047	0.050	0.048	0.047	0.050	0.053	0.046	1.048	0.000	0.044	0.043	0.043	0.040
F_i	0.085	0.145	0.073	0.088	0.083	0.088	0.084	0.093	0.096	0.096	0.098	0.093	0.091	1.000	0.117	0.115	0.114	0.109
P_a	1.321	1.337	0.996	1.349	1.223	1.324	1.213	1.446	1.589	1.591	1.645	1.483	1.442	0.000	2.338	1.296	1.290	1.377
P_{ax}	0.037	0.036	0.034	0.038	0.038	0.040	0.040	0.041	0.040	0.040	0.040	0.040	0.039	0.000	0.037	1.037	0.037	0.090
P_{fm}	0.111	0.110	0.092	0.111	0.103	0.120	0.104	0.152	0.118	0.119	0.123	0.139	0.118	0.000	0.109	0.107	1.138	0.107
P_m	0.723	0.707	0.666	0.747	0.744	0.784	0.775	0.795	0.776	0.775	0.768	0.783	0.755	0.000	0.720	0.724	0.729	1.755
P_s	1.045	1.009	1.024	1.100	1.089	1.087	1.110	1.103	1.078	1.087	1.088	1.085	1.076	0.000	1.027	1.140	1.096	1.235
P_{pdm}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.000	0.003	0.003	0.003	0.003
P_e	0.081	0.078	0.084	0.074	0.079	0.071	0.101	0.077	0.088	0.081	0.080	0.084	0.083	0.000	0.079	0.077	0.077	0.071
P_{he}	0.023	0.023	0.022	0.022	0.024	0.024	0.021	0.025	0.026	0.025	0.023	0.025	0.024	0.000	0.023	0.022	0.022	0.020
P_{fp}	0.001	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001
C_a	1.427	1.444	1.076	1.458	1.322	1.431	1.310	1.563	1.717	1.719	1.778	1.603	1.558	0.000	1.446	1.400	1.394	1.488
C_{ax}	0.038	0.037	0.035	0.039	0.039	0.041	0.041	0.042	0.041	0.041	0.041	0.041	0.040	0.000	0.038	0.038	0.038	0.092

C_{fm}	0.119	0.118	0.099	0.119	0.110	0.129	0.111	0.163	0.126	0.128	0.132	0.149	0.126	0.000	0.117	0.115	0.148	0.115
C_m	1.460	1.427	1.344	1.509	1.503	1.582	1.565	1.605	1.567	1.565	1.549	1.580	1.523	0.000	1.454	1.462	1.471	1.523
C_s	1.077	1.040	1.056	1.134	1.123	1.121	1.144	1.137	1.111	1.120	1.122	1.118	1.110	0.000	1.059	1.176	1.130	1.274
C_{padm}	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.000	0.003	0.003	0.003	0.003
C_e	0.081	0.078	0.084	0.074	0.079	0.071	0.101	0.077	0.088	0.081	0.080	0.084	0.083	0.000	0.079	0.077	0.077	0.071
C_{he}	0.023	0.023	0.022	0.022	0.024	0.024	0.021	0.025	0.026	0.025	0.023	0.025	0.024	0.000	0.023	0.022	0.022	0.020
C_{fp}	0.001	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001
C_{tm}	0.263	0.261	0.224	0.270	0.259	0.277	0.265	0.291	0.295	0.295	0.298	0.290	0.279	0.000	0.263	0.261	0.264	0.275
X_a	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X_{mi}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
X_m	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table A2. (Continued)

	[19]	[20]	[21]	[22]	[23]	[24]	[25]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	[33]	[34]	[35]	[36]
	P_s	P_{padm}	P_e	P_{he}	P_{fp}	C_a	C_{ax}	C_{fm}	C_m	C_s	C_{padm}	C_e	C_{he}	C_{fp}	C_{tm}	X_a	X_{mi}	X_m
F_L	1.443	1.690	1.643	1.635	1.635	1.506	1.484	1.483	0.799	1.400	1.690	1.643	1.634	1.634	1.400	1.518	1.489	1.320
F_C	1.846	1.468	1.679	1.669	1.669	1.913	1.888	1.873	1.079	1.791	1.468	1.679	1.668	1.668	1.791	1.927	1.879	1.804
H_{uk}	0.814	0.820	0.846	0.841	0.841	0.847	0.835	0.832	0.461	0.790	0.820	0.846	0.841	0.841	0.790	0.854	0.835	0.767
H_{us}	0.149	0.147	0.153	0.152	0.152	0.155	0.153	0.153	0.085	0.145	0.147	0.153	0.152	0.152	0.145	0.157	0.153	0.141
H_{uw}	0.086	0.084	0.088	0.087	0.087	0.090	0.089	0.088	0.049	0.084	0.084	0.088	0.087	0.087	0.084	0.091	0.089	0.082
H_{un}	0.061	0.060	0.063	0.062	0.062	0.064	0.063	0.062	0.035	0.059	0.060	0.063	0.062	0.062	0.059	0.064	0.063	0.058
H_{ue}	0.062	0.061	0.064	0.063	0.063	0.065	0.064	0.064	0.036	0.061	0.061	0.064	0.063	0.063	0.061	0.065	0.064	0.059
H_{rk}	0.050	0.048	0.050	0.050	0.050	0.052	0.051	0.051	0.028	0.048	0.048	0.050	0.050	0.050	0.048	0.052	0.051	0.047
H_{rs}	0.484	0.457	0.484	0.481	0.481	0.503	0.496	0.493	0.277	0.470	0.457	0.484	0.481	0.481	0.470	0.507	0.495	0.461
H_{rw}	0.551	0.519	0.550	0.547	0.548	0.573	0.565	0.562	0.315	0.535	0.519	0.550	0.547	0.547	0.535	0.577	0.564	0.525
H_{rn}	0.371	0.351	0.371	0.369	0.369	0.385	0.380	0.378	0.212	0.360	0.351	0.371	0.369	0.369	0.360	0.388	0.379	0.353
H_{re}	0.560	0.528	0.560	0.557	0.557	0.582	0.574	0.571	0.320	0.543	0.528	0.560	0.557	0.557	0.543	0.587	0.573	0.534

H_{ra}	0.042	0.040	0.042	0.042	0.042	0.043	0.043	0.043	0.024	0.041	0.040	0.042	0.042	0.042	0.041	0.044	0.043	0.040
F_i	0.112	0.089	0.102	0.101	0.101	0.116	0.114	0.114	0.065	0.109	0.089	0.102	0.101	0.101	0.109	0.117	0.114	0.109
P_a	1.278	1.281	1.326	1.300	1.300	2.254	1.292	1.287	0.810	1.239	1.281	1.326	1.299	1.299	1.239	2.338	1.290	1.377
P_{ax}	0.038	0.039	0.038	0.038	0.038	0.037	1.011	0.037	0.048	0.037	0.039	0.038	0.038	0.038	0.037	0.037	0.037	0.090
P_{fm}	0.105	0.101	0.105	0.105	0.105	0.108	0.107	1.069	0.064	0.101	0.101	0.105	0.105	0.105	0.101	0.109	1.138	0.107
P_m	0.737	0.758	0.734	0.736	0.736	0.718	0.722	0.728	0.943	0.715	0.758	0.734	0.736	0.736	0.715	0.720	0.729	1.755
P_s	2.155	1.238	1.203	1.212	1.212	1.102	1.160	1.162	0.828	2.091	1.238	1.203	1.211	1.211	2.091	1.027	1.096	1.235
$P_{\mu dm}$	0.003	1.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.003	1.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
P_e	0.075	0.073	1.076	0.076	0.076	0.078	0.077	0.076	0.043	0.073	0.073	1.076	0.076	0.076	0.073	0.079	0.077	0.071
P_{he}	0.022	0.021	0.022	1.022	0.022	0.022	0.022	0.022	0.012	0.021	0.021	0.022	1.021	0.022	0.021	0.023	0.022	0.020
P_{fp}	0.001	0.001	0.001	0.001	1.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001	1.000	0.001	0.001	0.001	0.001
C_a	1.3841	1.385	1.433	1.405	1.405	2.435	1.396	1.390	0.876	1.339	1.385	1.433	1.404	1.404	1.339	1.446	1.394	1.488
C_{ax}	0.039	0.040	0.039	0.039	0.039	0.038	1.038	0.038	0.049	0.038	0.040	0.039	0.039	0.039	0.038	0.038	0.038	0.092
C_{fm}	0.112	0.108	0.113	0.112	0.112	0.116	0.115	1.146	0.068	0.109	0.108	0.113	0.112	0.112	0.109	0.117	0.148	0.115
C_m	1.487	1.530	1.482	1.486	1.486	1.450	1.458	1.469	1.904	1.443	1.530	1.482	1.485	1.485	1.443	1.454	1.471	1.523
C_s	1.191	1.276	1.240	1.249	1.249	1.136	1.196	1.198	0.853	2.155	1.276	1.240	1.248	1.248	2.155	1.059	1.130	1.274
$C_{\mu dm}$	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.033	1.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
C_e	0.075	0.073	0.076	0.076	0.076	0.078	0.077	0.076	0.043	0.073	0.073	1.076	0.076	0.076	0.073	0.079	0.077	0.071
C_{he}	0.022	0.021	0.022	0.022	0.022	0.022	0.022	0.022	0.012	0.021	0.021	0.022	1.022	0.022	0.021	0.023	0.022	0.020
C_{fp}	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001	1.001	0.001	0.001	0.001	0.001
C_m	0.262	0.266	0.265	0.264	0.264	0.335	0.284	0.329	0.266	0.254	0.266	0.265	0.263	0.263	1.254	0.263	0.264	0.275
X_a	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
X_{mi}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
X_m	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Table A3. Longest Paths in M' ({})

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m
F_L	1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	1,10	1,11	1,12	1,11,13	1,2,14	1,15	1,18,16	1,12,17	1,18
F_C	2,1	2	2,3	2,4	2,5	2,1,6	2,1,7	2,8	2,9	2,10	2,11	2,12	2,11,13	2,14	2,15	2,18,16	2,12,17	2,18
H_{uk}	3,1	3,2	3	3,1,4	3,1,5	3,1,6	3,1,7	3,2,8	3,2,9	3,2,10	3,2,11	3,2,12	3,2,11,13	3,2,14	3,15	3,18,16	3,2,12,17	3,18
H_{us}	4,1	4,2	4,3	4	4,1,5	4,1,6	4,1,7	4,2,8	4,2,9	4,10	4,2,11	4,12	4,13	4,2,14	4,15	4,18,16	4,17	4,18
H_{uw}	5,1	5,2	5,3	5,1,4	5	5,1,6	5,1,7	5,2,8	5,2,9	5,2,10	5,2,11	5,2,12	5,13	5,2,14	5,15	5,18,16	5,2,12,17	5,18
H_{un}	6,1	6,2	6,3	6,1,4	6,1,5	6	6,1,7	6,2,8	6,2,9	6,10	6,2,11	6,12	6,13	6,2,14	6,15	6,18,16	6,17	6,18
H_{ue}	7,1	7,6	7,3	7,1,4	7,1,5	7,1,6	7	7,2,8	7,2,9	7,10	7,2,11	7,12	7,13	7,2,14	7,15	7,18,16	7,12,17	7,18
H_{rk}	8,1	8,2	8,3	8,1,4	8,1,5	8,1,6	8,1,7	8	8,9	8,10	8,2,11	8,12	8,13	8,2,14	8,15	8,18,16	8,17	8,18
H_{rs}	9,1	9,2	9,3	9,1,4	9,1,5	9,1,6	9,1,7	9,2,8	9	9,10	9,11	9,12	9,13	9,2,14	9,15	9,18,16	9,17	9,18
H_{rw}	10,1	10,2	10,3	10,1,4	10,1,5	10,1,6	10,1,7	10,2,8	10,9	10	10,11	10,12	10,13	10,2,14	10,15	10,18,16	10,17	10,18
H_{rn}	11,1	11,2	11,3	11,1,4	11,1,5	11,1,6	11,1,7	11,2,8	11,9	11,10	11	11,12	11,13	11,2,14	11,15	11,18,16	11,17	11,18
H_{re}	12,1	12,2	12,3	12,1,4	12,1,5	12,1,6	12,1,7	12,2,8	12,9	12,10	12,2,11	12	12,13	12,2,14	12,15	12,18,16	12,17	12,18
H_{tra}	13,1	13,2	13,3	13,4	13,5	13,6	13,7	13,8	13,9	13,10	13,11	13,12	13	13,2,14	13,15	13,18,16	13,17	13,18
F_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
P_a	15,1	15,2	15,3	15,4	15,5	15,1,6	15,1,7	15,8	15,9	15,10	15,11	15,12	15,11,13	15,2,14	15	15,18,16	15,12,17	15,18
P_{ax}	16,1	16,2	16,3	16,4	16,5	16,1,6	16,1,7	16,2,8	16,9	16,10	16,11	16,12	16,11,13	16,2,14	16,15	16	16,12,17	16,18
P_{fm}	17,1	17,2	17,3	17,4	17,5	17,1,6	17,1,7	17,2,8	17,9	17,10	17,11	17,12	17,11,13	17,2,14	17,15	17,18,16	17	17,18
P_m	18,1	18,2	18,3	18,1,4	18,1,5	18,1,6	18,1,7	18,2,8	18,9	18,10	18,11	18,12	18,11,13	18,2,14	18,15	18,16	18,12,17	18
P_s	19,1	19,2	19,3	19,4	19,1,5	19,1,6	19,1,7	19,2,8	19,9	19,10	19,11	19,12	19,11,13	19,2,14	19,15	19,18,16	19,12,17	19,18
P_{pdm}	20,1	20,2	20,3	20,1,4	20,1,5	20,1,6	20,1,7	20,1,8	20,9	20,10	20,11	20,12	20,11,13	20,2,14	20,15	20,18,16	20,12,17	20,18
P_e	21,1	21,2	21,3	21,4	21,1,5	21,1,6	21,1,7	21,1,8	21,9	21,10	21,11	21,12	21,11,13	21,2,14	21,15	21,18,16	21,12,17	21,18
P_{he}	22,1	22,2	22,3	22,4	22,1,5	22,1,6	22,1,7	22,1,8	22,9	22,10	22,11	22,12	22,11,13	22,2,14	22,15	22,18,16	22,12,17	22,18
P_{fp}	23,1	23,2	23,3	23,4	23,1,5	23,1,6	23,1,7	23,1,8	23,9	23,10	23,11	23,12	23,11,13	23,2,14	23,15	23,18,16	23,12,17	23,18
C_a	24,1	24,2	24,3	24,4	24,5	24,1,6	24,1,7	24,8	24,9	24,10	24,11	24,12	24,11,13	24,2,14	24,15	24,18,16	24,12,17	24,18
C_{ax}	25,1	25,2	25,3	25,4	25,5	25,1,6	25,1,7	25,2,8	25,9	25,10	25,11	25,12	25,11,13	25,2,14	25,15	25,16	25,12,17	25,18

C_{fm}	26,1	26,2	26,3	26,4	26,5	26,1,6	26,1,7	26,2,8	26,9	26,10	26,11	26,12	26,11,13	26,2,14	26,15	26,18,16	26,17	26,18
C_m	27,1	27,2	27,3	27,1,4	27,1,5	27,1,6	27,1,7	27,2,8	27,2,9	27,2,10	27,2,11	27,2,12	27,2,11,13	27,2,14	27,15	27,18,16	27,2,12,17	27,18
C_s	28,1	28,2	28,3	28,1,4	28,1,5	28,1,6	28,1,7	28,2,8	28,9	28,10	28,11	28,12	28,11,13	28,2,14	28,15	28,18,16	28,12,17	28,18
C_{padm}	29,1	29,2	29,3	29,1,4	29,1,5	29,1,6	29,1,7	29,1,8	29,9	29,10	29,11	29,12	29,11,13	29,2,14	29,15	29,18,16	29,12,17	29,18
C_e	30,1	30,2	30,3	30,4	30,1,5	30,1,6	30,1,7	30,1,8	30,9	30,10	30,11	30,12	30,11,13	30,2,14	30,15	30,18,16	30,12,17	30,18
C_{he}	31,1	31,2	31,3	31,4	31,1,5	31,1,6	31,1,7	31,1,8	31,9	31,10	31,11	31,12	31,11,13	31,2,14	31,15	31,18,16	31,12,17	31,18
C_{fp}	32,1	32,2	32,3	32,4	32,1,5	32,1,6	32,1,7	32,2,8	32,9	32,10	32,11	32,12	32,11,13	32,2,14	32,15	32,18,16	32,12,17	32,18
C_{tm}	33,1	33,2	33,3	33,1,4	33,1,5	33,1,6	33,1,7	33,2,8	33,9	33,10	33,11	33,12	33,11,13	33,2,14	33,15	33,18,16	33,12,17	33,18
X_a	34,1	34,2	34,3	34,4	34,5	34,1,6	34,1,7	34,8	34,9	34,10	34,11	34,12	34,11,13	34,2,14	34,15	34,18,16	34,12,17	34,18
X_{mi}	35,1	35,2	35,3	35,4	35,5	35,1,6	35,1,7	35,2,8	35,9	35,10	35,11	35,12	35,11,13	35,2,15	35,15	35,18,16	35,17	35,18
X_m	36,1	36,2	36,3	36,1,4	36,1,5	36,1,6	36,1,7	36,2,8	36,9	36,10	36,11	36,12	36,11,13	36,2,14	36,15	36,16	36,12,17	36,18

Table A3. (Continued)

	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
F_L	1,19	1,5,20	1,21	1,9,22	1,9,23	1,24	1,18,25	1,26	1,27	1,28	1,5,29	1,30	1,9,31	1,9,32	1,24,33	34	35	36
F_C	2,19	2,5,20	2,21	2,9,22	2,9,23	2,24	2,18,25	2,12,26	2,27	2,28	2,5,29	2,30	2,9,31	2,9,32	2,24,33	34	35	36
H_{uk}	3,19	3,1,5,20	3,21	3,2,9,22	3,2,9,23	3,24	3,18,25	3,2,12,26	3,27	3,28	3,1,5,29	3,30	3,2,9,31	3,2,9,32	3,24,33	34	35	36
H_{us}	4,19	4,1,5,20	4,1,21	4,2,9,22	4,2,9,23	4,24	4,18,25	4,26	4,27	4,28	4,1,5,29	4,1,30	4,2,9,31	4,2,9,32	4,24,33	34	35	36
H_{uw}	5,19	5,20	5,21	5,2,9,22	5,2,9,23	5,24	5,18,25	5,26	5,27	5,28	5,29	5,30	5,2,9,31	5,2,9,32	5,24,33	34	35	36
H_{un}	6,19	6,20	6,1,21	6,22	6,2,9,23	6,24	6,18,25	6,26	6,27	6,28	6,29	6,1,30	6,31	6,2,9,32	6,33	34	35	36
H_{ue}	7,19	7,1,5,20	7,21	7,2,9,22	7,2,9,23	7,24	7,18,25	7,26	7,27	7,28	7,1,5,29	7,30	7,2,9,31	7,2,9,32	7,24,33	34	35	36
H_{rk}	8,19	8,1,8,20	8,21	8,22	8,9,23	8,24	8,18,25	8,26	8,27	8,28	8,1,5,29	8,30	8,31	8,9,32	8,33	34	35	36
H_{rs}	9,19	9,1,5,20	9,21	9,22	9,23	9,24	9,18,25	9,26	9,27	9,28	9,1,5,29	9,30	9,31	9,32	9,33	34	35	36
H_{rw}	10,19	10,1,5,20	10,21	10,22	10,9,23	10,24	10,18,25	10,26	10,27	10,28	10,1,5,29	10,30	10,31	10,9,32	10,33	34	35	36
H_{rn}	11,19	11,1,5,20	11,21	11,9,22	11,9,23	11,24	11,18,25	11,26	11,27	11,28	11,1,5,29	11,30	11,9,31	11,9,32	11,33	34	35	36
H_{re}	12,19	12,1,5,20	12,21	12,22	12,9,23	12,24	12,18,25	12,26	12,27	12,28	12,1,5,29	12,30	12,31	12,9,32	12,33	34	35	36

H_{tra}	13,19	13,5,20	13,21	13,9,22	13,9,23	13,24	13,18,25	13,26	13,27	13,28	13,5,29	13,30	13,9,31	13,9,32	13,33	34	35	36
F_i	19	20	21	22,	23	24	25	26	27	28	29	30	31	32	33	34	35	36
P_a	15,19	15,5,20	15,21	15,9,22	15,9,23	15,24	15,18,25	15,12,26	15,27	15,28	15,5,29	15,30	15,9,31	15,9,32	15,24,33	34	35	36
P_{ax}	16,19	16,5,20	16,1,21	16,9,22	16,9,23	16,24	16,18,25	16,12,26	16,27	16,28	16,5,29	16,1,30	16,9,31	16,9,32	16,24,33	34	35	36
P_{fm}	17,19	17,5,20	17,1,21	17,9,22	17,9,23	17,24	17,18,25	17,26	17,27	17,28	17,5,29	17,1,30	17,9,31	17,9,32	17,24,33	34	35	36
P_m	18,19	18,1,5,20	18,1,21	18,9,22	18,9,23	18,24	18,25	18,12,26	18,27	18,28	18,1,5,29	18,1,30	18,9,31	18,9,32	18,33	34	35	36
P_s	19	19,1,5,20	19,1,21	19,9,22	19,9,23	19,24	19,18,25	19,12,26	19,27	19,28	19,1,5,29	19,1,30	19,9,31	19,9,32	19,24,33	34	35	36
P_{pdm}	20,19	20	20,1,21	20,9,22	20,9,23	20,24	20,18,25	20,12,26	20,27	20,28	20,1,5,29	20,1,30	20,9,31	20,9,32	20,24,33	34	35	36
P_e	21,19	21,1,5,20	21	21,9,22	21,9,23	21,24	21,18,25	21,12,26	21,27	21,28	21,1,5,29	21,1,30	21,9,31	21,9,32	21,24,33	34	35	36
P_{he}	22,19	22,1,5,20	22,1,21	22	22,9,23	22,24	22,18,25	22,12,26	22,27	22,28	22,1,5,29	22,1,30	22,9,31	22,9,32	22,24,33	34	35	36
P_{fp}	23,19	23,1,5,20	23,1,21	23,9,22	23	23,24	23,18,25	23,12,26	23,27	23,28	23,1,5,29	23,1,30	23,9,31	23,9,32	23,24,33	34	35	36
C_a	24,19	24,5,20	24,21	24,9,22	24,9,23	24	24,18,25	24,12,26	24,27	24,28	24,5,29	24,30	24,9,31	24,9,32	24,33	34	35	36
C_{ax}	25,19	25,5,20	25,1,21	25,9,22	25,9,23	25,24	25	25,12,26	25,27	25,28	25,5,29	25,1,30	25,9,31	25,9,32	25,33	34	35	36
C_{fm}	26,19	26,5,20	26,1,21	26,9,22	26,9,23	26,24	26,18,25	26	26,27	26,28	26,5,29	26,1,30	26,9,31	26,9,32	26,33	34	35	36
C_m	27,19	27,1,5,20	27,1,21	27,2,9,22	27,2,9,23	27,24	27,18,25	27,2,12,26	27	27,28	27,1,5,29	27,1,30	27,2,9,31	27,2,9,32	27,33	34	35	36
C_s	28,19	28,1,5,20	28,1,21	28,9,22	28,9,23	28,24	28,18,25	28,12,26	28,27	28	28,1,5,29	28,1,30	28,9,31	28,9,32	28,24,33	34	35	36
C_{pdm}	29,19	29,20	29,1,21	29,9,22	29,9,23	29,24	29,18,25	29,12,26	29,27	29,28	29	29,1,30	29,9,31	29,9,32	29,24,33	34	35	36
C_e	30,19	30,1,5,20	302,21	30,9,22	30,9,23	30,24	30,18,25	30,12,26	30,27	30,28	30,1,5,29	30	30,9,31	30,9,32	30,24,33	34	35	36
C_{he}	31,19	31,1,5,20	31,1,21	31,22	31,9,23	31,24	31,18,25	31,12,26	31,27	31,28	31,1,5,29	31,1,30	31	31,9,32	31,24,33	34	35	36
C_{fp}	32,19	32,1,5,20	32,1,21	32,9,22	32,23	32,24	32,18,25	32,12,26	32,27	32,28	32,1,5,29	32,1,30	32,9,31	32	32,24,33	34	35	36
C_{tm}	33,19	33,1,5,20	33,1,21	33,9,22	33,9,23	33,24	33,18,25	33,12,26	33,27	33,28	33,1,5,29	33,1,30	33,9,31	33,9,32	33	34	35	36
X_a	34,19	34,5,20	34,21	34,9,22	34,9,23	34,24	34,18,25	34,12,26	34,27	34,28	34,5,29	34,30	34,9,31	34,9,32	34,24,33	34	35	36
X_{mi}	35,19	35,5,20	35,1,21	35,9,22	35,9,23	35,24	35,18,25	35,26	35,27	35,28	35,5,29	35,1,30	35,9,31	35,9,32	35,24,33	34	35	36
X_m	36,19	36,1,5,20	36,1,21	36,9,22	36,9,23	36,24	36,25	36,12,26	36,27	36,28	36,1,5,29	36,1,30	36,9,31	36,9,32	36,33	34	35	36

Table A4. Longest Path Influence in M'

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m
F_L	1.00	1.41	0.94	0.17	0.09	0.07	0.07	0.05	0.05	0.57	0.39	0.58	0.02	0.02	1.32	0.06	0.08	0.72
F_C	1.09	1.00	0.81	0.15	0.09	0.07	0.07	0.05	0.52	0.60	0.40	0.61	0.02	0.15	1.34	0.06	0.08	0.71
H_{uk}	0.94	1.20	1.00	0.16	0.09	0.06	0.06	0.06	0.63	0.72	0.48	0.73	0.03	0.17	1.00	0.06	0.10	0.67
H_{us}	1.13	1.45	0.64	1.00	0.11	0.08	0.08	0.08	0.76	0.44	0.58	0.44	0.06	0.21	1.35	0.07	0.11	0.75
H_{uw}	1.07	1.37	0.61	0.18	11.00	0.07	0.07	0.07	0.72	0.82	0.55	0.83	0.05	0.20	1.22	0.07	0.12	0.74
H_{un}	1.12	1.44	0.64	0.19	0.11	1.00	0.08	0.08	0.76	0.43	0.58	0.44	0.05	0.21	1.32	0.07	0.12	0.78
H_{ue}	1.09	1.39	0.62	0.18	0.10	0.07	1.00	0.07	0.73	0.42	0.56	0.43	0.05	0.20	1.21	0.07	0.06	0.78
H_{rk}	1.19	1.53	0.68	0.20	0.11	0.08	0.08	1.00	0.40	0.46	0.61	0.47	0.05	0.22	1.45	0.07	0.15	0.80
H_{rs}	1.24	1.58	0.70	0.20	0.12	0.08	0.08	0.08	1.00	0.47	0.32	0.48	0.05	0.23	1.59	0.07	0.12	0.78
H_{rw}	1.24	1.58	0.70	0.20	0.12	0.08	0.08	0.08	0.42	1.00	0.32	0.48	0.05	0.23	1.59	0.07	0.12	0.78
H_{rn}	1.26	1.61	0.71	0.21	0.12	0.09	0.09	0.09	0.42	0.48	1.00	0.49	0.05	0.23	1.65	0.07	0.12	0.77
H_{re}	1.20	1.54	0.68	0.20	0.11	0.08	0.08	0.08	0.40	0.46	0.61	1.00	0.05	0.22	1.48	0.07	0.14	0.78
H_{tra}	1.17	1.49	0.84	0.18	0.09	0.07	0.07	0.06	0.52	0.67	0.48	0.63	1.00	0.22	1.44	0.07	0.12	0.75
F_i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_a	1.52	1.93	0.85	0.16	0.09	0.10	0.10	0.05	0.51	0.58	0.39	0.59	0.02	0.28	1.00	0.06	0.08	0.72
P_{ax}	1.49	1.89	0.84	0.15	0.09	0.10	0.10	0.10	0.50	0.57	0.38	0.58	0.02	0.27	1.30	1.00	0.08	0.72
P_{fm}	1.49	1.88	0.83	0.15	0.09	0.10	0.10	0.10	0.50	0.56	0.38	0.57	0.02	0.27	1.29	0.07	1.00	0.73
P_m	1.32	1.80	0.77	0.22	0.12	0.09	0.09	0.10	0.46	0.53	0.35	0.53	0.02	0.26	1.38	0.09	0.07	1.00
P_s	1.44	1.85	0.81	0.15	0.14	0.10	0.10	0.10	0.48	0.55	0.37	0.56	0.02	0.27	1.28	0.07	0.08	0.74
P_{pdm}	1.69	1.47	0.82	0.28	0.16	0.11	0.12	0.09	0.46	0.52	0.35	0.53	0.02	0.21	1.28	0.07	0.07	0.76
P_e	1.64	1.68	0.85	0.15	0.15	0.11	0.11	0.09	0.48	0.55	0.37	0.56	0.02	0.24	1.33	0.07	0.08	0.73
P_{he}	1.64	1.67	0.84	0.15	0.15	0.11	0.11	0.09	0.48	0.55	0.37	0.56	0.02	0.24	1.30	0.07	0.08	0.74
P_{fp}	1.64	1.67	0.84	0.15	0.15	0.11	0.11	0.09	0.48	0.55	0.37	0.56	0.02	0.24	1.30	0.07	0.08	0.74
C_a	1.51	1.91	0.85	0.16	0.09	0.10	0.10	0.05	0.50	0.57	0.39	0.58	0.02	0.28	2.25	0.06	0.08	0.72
C_{ax}	1.48	1.89	0.84	0.15	0.09	0.10	0.10	0.10	0.50	0.56	0.38	0.57	0.02	0.27	1.29	1.01	0.08	0.72

C_{fm}	1.48	1.87	0.83	0.15	0.09	0.10	0.10	0.10	0.49	0.56	0.38	0.57	0.02	0.27	1.29	0.07	1.07	0.73
C_m	0.80	1.08	0.46	0.13	0.07	0.05	0.05	0.06	0.57	0.65	0.43	0.65	0.02	0.16	0.81	0.08	0.09	0.94
C_s	1.40	1.79	0.79	0.23	0.13	0.09	0.10	0.09	0.47	0.53	0.36	0.54	0.02	0.26	1.24	0.06	0.08	0.71
C_{padm}	1.69	1.47	0.82	0.28	0.16	0.11	0.12	0.09	0.46	0.52	0.35	0.53	0.02	0.21	1.28	0.07	0.07	0.76
C_e	1.64	1.68	0.85	0.15	0.15	0.11	0.11	0.09	0.48	0.55	0.37	0.56	0.02	0.24	1.33	0.07	0.08	0.73
C_{he}	1.63	1.67	0.84	0.15	0.15	0.11	0.11	0.09	0.48	0.55	0.37	0.56	0.02	0.24	1.30	0.07	0.08	0.74
C_{fp}	1.63	1.67	0.84	0.15	0.15	0.11	0.11	0.09	0.48	0.55	0.37	0.56	0.02	0.24	1.30	0.07	0.08	0.74
C_{tm}	1.40	1.79	0.79	0.23	0.13	0.09	0.10	0.09	0.47	0.53	0.36	0.54	0.02	0.26	1.24	0.06	0.08	0.71
X_a	1.52	1.93	0.85	0.16	0.09	0.10	0.10	0.05	0.51	0.58	0.39	0.59	0.02	0.28	2.34	0.06	0.08	0.72
X_{mi}	1.49	1.88	0.83	0.15	0.09	0.10	0.10	0.10	0.50	0.56	0.38	0.57	0.02	0.27	1.29	0.07	1.14	0.73
X_m	1.32	1.80	0.77	0.22	0.12	0.09	0.09	0.10	0.46	0.53	0.35	0.53	0.02	0.26	1.38	0.09	0.07	1.75

Table A4. (Continued)

	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
F_L	1.04	0.00	0.08	0.01	0.00	1.43	0.07	0.12	1.46	1.08	0.00	0.08	0.01	0.00	0.48	1.00	1.00	1.00
F_C	1.01	0.00	0.08	0.01	0.00	1.44	0.06	0.09	1.43	1.04	0.00	0.08	0.01	0.00	0.48	1.00	1.00	1.00
H_{uk}	1.02	0.00	0.08	0.02	0.00	1.08	0.06	0.11	1.34	1.06	0.00	0.08	0.02	0.00	0.36	1.00	1.00	1.00
H_{us}	1.10	0.00	0.09	0.02	0.00	1.46	0.07	0.12	1.51	1.13	0.00	0.09	0.02	0.00	0.49	1.00	1.00	1.00
H_{uw}	1.09	0.00	0.08	0.02	0.00	1.32	0.07	0.11	1.50	1.12	0.00	0.08	0.02	0.00	0.44	1.00	1.00	1.00
H_{un}	1.09	0.00	0.09	0.02	0.00	1.43	0.07	0.13	1.58	1.12	0.00	0.09	0.02	0.00	0.28	1.00	1.00	1.00
H_{ue}	1.11	0.00	0.10	0.02	0.00	1.31	0.07	0.11	1.56	1.14	0.00	0.10	0.02	0.00	0.44	1.00	1.00	1.00
H_{rk}	1.10	0.00	0.08	0.03	0.00	1.56	0.07	0.16	1.61	1.14	0.00	0.08	0.03	0.00	0.29	1.00	1.00	1.00
H_{rs}	1.08	0.00	0.09	0.03	0.00	1.72	0.07	0.13	1.57	1.11	0.00	0.09	0.03	0.00	0.30	1.00	1.00	1.00
H_{rw}	1.09	0.00	0.08	0.02	0.00	1.72	0.07	0.13	1.56	1.12	0.00	0.08	0.02	0.00	0.30	1.00	1.00	1.00
H_{rn}	1.09	0.00	0.08	0.01	0.00	1.78	0.07	0.13	1.55	1.12	0.00	0.08	0.01	0.00	0.30	1.00	1.00	1.00
H_{re}	1.08	0.00	0.08	0.02	0.00	1.60	0.07	0.15	1.58	1.12	0.00	0.08	0.02	0.00	0.29	1.00	1.00	1.00

H_{tra}	1.08	0.00	0.08	0.01	0.00	1.56	0.07	0.13	1.52	1.11	0.00	0.08	0.01	0.00	0.28	1.00	1.00	1.00
F_i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
P_a	1.03	0.00	0.08	0.01	0.00	1.45	0.07	0.09	1.45	1.06	0.00	0.08	0.01	0.00	0.48	1.00	1.00	1.00
P_{ax}	1.14	0.00	0.12	0.01	0.00	1.40	0.07	0.09	1.46	1.18	0.00	0.12	0.01	0.00	0.47	1.00	1.00	1.00
P_{fm}	1.10	0.00	0.12	0.01	0.00	1.39	0.07	0.15	1.47	1.13	0.00	0.12	0.01	0.00	0.47	1.00	1.00	1.00
P_m	1.24	0.00	0.11	0.01	0.00	1.49	0.09	0.08	1.52	1.27	0.00	0.11	0.01	0.00	0.27	1.00	1.00	1.00
P_s	1.00	0.00	0.12	0.01	0.00	1.38	0.07	0.08	1.49	1.19	0.00	0.12	0.01	0.00	0.46	1.00	1.00	1.00
P_{pdm}	1.24	1.00	0.14	0.01	0.00	1.38	0.07	0.08	1.53	1.28	0.00	0.14	0.01	0.00	0.46	1.00	1.00	1.00
P_e	1.20	0.00	1.00	0.01	0.00	1.43	0.07	0.08	1.48	1.24	0.00	0.13	0.01	0.00	0.48	1.00	1.00	1.00
P_{he}	1.21	0.00	0.13	1.00	0.00	1.40	0.07	0.08	1.49	1.25	0.00	0.13	0.01	0.00	0.47	1.00	1.00	1.00
P_{fp}	1.21	0.00	0.13	0.01	1.00	1.40	0.07	0.08	1.49	1.25	0.00	0.13	0.01	0.00	0.47	1.00	1.00	1.00
C_a	1.10	0.00	0.08	0.01	0.00	1.00	0.07	0.09	1.45	1.14	0.00	0.08	0.01	0.00	0.33	1.00	1.00	1.00
C_{ax}	1.16	0.00	0.12	0.01	0.00	1.40	1.00	0.09	1.46	1.20	0.00	0.12	0.01	0.00	0.28	1.00	1.00	1.00
C_{fm}	1.16	0.00	0.12	0.01	0.00	1.39	0.07	1.00	1.47	1.20	0.00	0.12	0.01	0.00	0.33	1.00	1.00	1.00
C_m	0.83	0.00	0.06	0.01	0.00	0.88	0.09	0.10	1.00	0.85	0.00	0.06	0.01	0.00	0.27	1.00	1.00	1.00
C_s	2.09	0.00	0.11	0.01	0.00	1.34	0.07	0.08	1.44	1.00	0.00	0.11	0.01	0.00	0.45	1.00	1.00	1.00
C_{pdm}	1.24	1.00	0.14	0.01	0.00	1.38	0.07	0.08	1.53	1.28	1.00	0.14	0.01	0.00	0.46	1.00	1.00	1.00
C_e	1.20	0.00	1.08	0.01	0.00	1.43	0.07	0.08	1.48	1.24	0.00	1.00	0.01	0.00	0.48	1.00	1.00	1.00
C_{he}	1.21	0.00	0.13	1.02	0.00	1.40	0.07	0.08	1.49	1.25	0.00	0.13	1.00	0.00	0.47	1.00	1.00	1.00
C_{fp}	1.21	0.00	0.13	0.01	1.00	1.40	0.07	0.08	1.49	1.25	0.00	0.13	0.01	1.00	0.47	1.00	1.00	1.00
C_{tm}	2.09	0.00	0.11	0.01	0.00	1.34	0.07	0.08	1.44	2.16	0.00	0.11	0.01	0.00	1.00	1.00	1.00	1.00
X_a	1.03	0.00	0.08	0.01	0.00	1.45	0.07	0.09	1.45	1.06	0.00	0.08	0.01	0.00	0.48	1.00	1.00	1.00
X_{mi}	1.10	0.00	0.12	0.01	0.00	1.39	0.07	0.15	1.47	1.13	0.00	0.12	0.01	0.00	0.47	1.00	1.00	1.00
X_m	1.24	0.00	0.11	0.01	0.00	1.49	0.09	0.08	1.52	1.27	0.00	0.11	0.01	0.00	0.27	1.00	1.00	1.00

Table A5. Shortest Paths in M' ({})

	[1] F_L	[2] F_C	[3] H_{uk}	[4] H_{us}	[5] H_{uw}	[6] H_{un}	[7] H_{ue}	[8] H_{rk}	[9] H_{rs}	[10] H_{rw}	[11] H_{rn}	[12] H_{re}	[13] H_{tra}	[14] F_i	[15] P_a	[16] P_{ax}	[17] P_{fm}	[18] P_m
F_L	1	1,2	1,32,7,3	1,20,4	1,20,5	1,32,6	1,32,7	1,32,8	1,32,7,9	1,32,7,10	1,32,7,11	1,32,7,12	1,32,13	1,14,2	1,32,7,15	1,16	1,29,17	1,18
F_C	2,1	2	2,20,7,3	2,20,4	2,20,5	2,6	2,20,7	2,32,8	2,20,7,9	2,20,7,10	2,20,7,11	2,20,7,12	2,32,13	2,20,14	2,20,7,15	2,16	2,29,17	2,18
H_{uk}	3,1	3,2	3,4,3	3,4	3,5	3,6	3,7	3,8	3,9	3,10	3,11	3,12	3,13	3,14	3,15	3,16	3,17	3,18
H_{us}	4,1	4,7,2	5,3	4	4,5	4,6	4,7	4,8	4,9	4,10	4,11	4,12	4,32,13	4,14	4,7,15	4,16	4,29,17	4,32,18
H_{uw}	5,1	5,2	6,3	5,4	5	5,6	5,7	5,8	5,9	5,10	5,11	5,12	5,32,13	5,14	5,15	5,16	5,17	5,32,18
H_{un}	6,1	6,7,2	7,3	6,4	6,5	6	6,7	6,8	6,9	6,10	6,11	6,12	6,32,13	6,14	6,7,15	6,32,16	6,29,17	6,32,18
H_{ue}	7,1	7,2	8,7,3	7,4	7,5	7,6	7	7,8	7,9	7,10	7,11	7,12	7,32,13	7,14	7,15	7,32,16	7,17	7,32,18
H_{rk}	8,7,1	8,7,2	9,7,3	8,4	8,5	8,6	8,7	8	8,9	8,10	8,11	8,12	8,32,13	8,20,14	8,7,15	8,32,16	8,29,17	8,32,18
H_{rs}	9,7,1	9,7,2	10,7,3	9,4	9,5	9,6	9,7	9,8	9	9,7,10	9,11	9,7,12	9,32,13	9,20,14	9,7,15	9,32,16	9,29,17	9,32,18
H_{rw}	10,7,1	10,7,2	11,7,3	10,4	10,5	10,6	10,7	10,8	10,9	10	10,11	10,7,12	10,32,13	10,20,14	10,7,15	10,32,16	10,29,17	10,32,18
H_{rn}	11,7,1	11,7,2	12,7,3	11,4	11,5	11,6	11,7	11,8	11,7,9	11,7,10	11,	11,7,12	11,32,13	11,20,14	11,7,15	11,32,16	11,29,17	11,32,18
H_{re}	12,7,1	12,7,2	12,7,3	12,4	12,5	12,6	12,7	12,8	12,9	12,10	12,11	12	12,32,13	12,20,14	12,7,15	12,32,16	12,29,17	12,32,18
H_{tra}	13,32,7,1	13,32,7,2	13,32,7,3	13,20,4	13,20,5	13,32,6	13,32,7	13,32,8	13,32,7,9	13,32,7,10	13,32,7,11	13,32,7,12	13	13,14	13,32,7,15	13,32,16	13,29,17	13,32,18
F_i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
P_a	15,20,7,1	15,20,7,2	15,20,7,3	15,20,4	15,20,5	15,32,6	15,20,7	15,32,8	15,20,7,9	15,20,7,10	15,20,7,11	15,20,7,12	15,32,13	15,20,14	15	15,16	15,29,17	15,18
P_{ax}	16,20,7,1	16,20,7,2	16,20,7,3	16,20,4	16,20,5	16,32,6	16,20,7	16,32,8	16,20,7,9	16,20,7,10	16,20,7,11	16,20,7,12	16,32,13	16,20,14	16,20,7,15	16	16,29,17	16,18
P_{fm}	17,20,7,1	17,20,7,2	17,20,7,3	17,20,4	17,20,5	17,6	17,20,7	17,32,8	17,20,7,9	17,20,7,10	17,20,7,11	17,20,7,12	17,32,13	17,20,14	17,20,15	17,16	17	17,18
P_m	18,7,1	18,7,2	18,7,3	18,4	18,5	18,6	18,7	18,8	18,7,9	18,7,10	18,7,11	18,7,12	18,13	18,20,14	18,7,15	18,32,16	18,29,17	18
P_s	19,7,1	19,7,2	19,7,3	19,4	19,5	19,6	19,7	19,8	19,7,9	19,7,10	19,7,11	19,7,12	19,13	19,20,14	19,7,15	19,16	19,29,17	19,32,18
P_{pdm}	20,7,1	20,7,2	20,7,3	20,4	20,5	20,6	20,7	20,8	20,7,9	20,7,10	20,7,11	20,7,12	20,13	20,14	20,7,15	20,32,16	20,17	20,32,18
P_e	21,7,1	21,7,2	21,7,3	21,20,4	21,20,5	21,6	21,7	21,8	21,7,9	21,7,10	21,7,11	21,7,12	21,13	21,20,14	21,7,15	21,16	21,29,17	21,18
P_{he}	22,7,1	22,7,2	22,7,3	22,20,4	22,20,5	22,6	22,7	22,8	22,7,9	22,7,10	22,7,11	22,7,12	22,13	22,20,14	22,7,15	22,16	22,29,17	22,18
P_{fp}	23,7,1	23,7,2	23,7,3	23,20,4	23,20,5	23,6	23,7	23,8	23,7,9	23,7,10	23,7,11	23,7,12	23,13	23,20,14	23,7,15	23,16	23,29,17	23,18
C_a	24,20,7,1	24,20,7,2	24,20,7,3	24,20,4	24,20,5	24,32,6	24,20,7	23,32,8	24,20,7,9	24,20,7,10	24,20,7,11	24,20,7,12	24,32,13	24,20,14	24,20,7,15	24,16	24,29,17	24,18
C_{ax}	25,20,7,1	25,20,7,2	25,20,7,3	25,20,4	25,20,5	25,6	25,20,7	25,32,8	25,20,7,9	25,20,7,10	25,20,7,11	25,20,7,12	25,32,13	25,20,14	25,20,7,15	25,32,16	25,29,17	25,18

C_{fm}	26,7,1	26,7,2	26,7,3	26,20,4	26,20,5	26,6	26,7	26,20,8	26,7,9	26,7,10	26,7,11	26,7,12	26,32,13	26,20,14	26,7,15	26,16	26,29,17	26,18
C_m	27,1	27,2	27,3	27,4	27,5	27,6	27,7	27,8	27,9	27,10	27,11,	27,12	27,13	27,14	27,15	27,32,16	27,17	27,32,18
C_s	28,7,1	28,7,2	28,7,3	28,4	28,5	28,6	28,7	28,8	28,7,9	28,7,10	28,7,11	28,7,12	28,13	28,20,14	28,15	28,16	28,17	28,18
C_{padm}	29,7,1	29,7,2	29,7,3	29,4	29,5	29,6	29,7	29,8	29,7,9	29,7,10	29,7,11	29,7,12	29,13	29,14	29,7,15	29,32,16	29,17	29,32,18
C_e	30,7,1	30,7,2	30,7,3	30,20,4	30,20,5	30,6	30,7	30,8	30,7,9	30,7,10	30,7,11	30,7,12	30,13	30,20,14	30,7,15	30,16	30,29,17	30,18
C_{he}	31,7,1	31,7,2	31,7,3	31,20,4	31,20,5	31,6	31,7	31,8	31,7,9	31,7,10	31,7,11	31,71,2	31,13	31,20,14	31,7,15	31,16	31,29,17	31,18
C_{fp}	32,7,1	32,7,2	32,7,3	32,20,4	32,20,5	32,6	32,7	32,8	32,7,9	32,7,10	32,7,11	32,7,12	32,13	32,20,14	32,7,15	32,16	32,29,17	32,18
C_{tm}	33,7,1	33,7,2	33,7,3	33,4	33,5	33,6	33,7	33,8	33,7,9	33,7,10	33,7,11	33,7,12	33,13	33,20,14	33,15	33,16	33,17	33,18
X_a	34,20,7,1	34,20,7,2	34,20,7,3	34,20,4	34,20,5	34,32,6	34,20,7	34,32,8	34,20,7,9	34,20,7,10	34,20,7,11	34,20,7,12	34,32,13	34,20,14	34,20,7,15	34,16	34,29,7	34,18
X_{mi}	35,20,7,1	35,20,7,2	35,20,7,3	35,20,4	35,20,5	35,6	35,20,7	35,32,8	35,20,7,9	35,20,7,10	35,20,7,11	35,20,7,12	35,32,13	35,20,14	35,20,7,15	35,16	35,29,17	35,18
X_m	36,7,1	36,7,2	36,7,3	36,4	36,5	36,6	36,7	36,8	36,7,9	36,7,10	36,7,11	36,7,12	36,13	36,20,14	36,7,15	36,32,16	36,29,17	36,32,18

Table A5. (Continued)

	[19] P_s	[20] P_{padm}	[21] P_e	[22] P_{he}	[23] P_{fp}	[24] C_a	[25] C_{ax}	[26] C_{fm}	[27] C_m	[28] C_s	[29] C_{padm}	[30] C_e	[31] C_{he}	[32] C_{fp}	[33] C_{tm}	[34] X_a	[35] X_{mi}	[36] X_m
F_L	1,19	1,20	1,20,21	1,32,22	1,23	1,32,7,24	1,25	1,20,26	1,27	1,28	1,29	1,29,30	1,32,31	1,32	1,33	34	35	36
F_C	2,19	2,20	2,20,21	2,32,22	2,23	2,20,7,24	2,25	2,20,26	2,27	2,28	2,29	2,29,30	2,32,31	2,32	2,33	34	35	36
H_{uk}	3,19	3,20	3,20,21	3,32,22	3,23	3,24	3,25	3,26	3,27	3,28	3,29	3,29,30	3,32,31	3,32	3,33	34	35	36
H_{us}	4,19	4,20	4,21	4,22	4,23	4,7,24	4,25	4,20,26	4,32,27	4,28	4,29	4,30	4,31	4,32	4,32,33	34	35	36
H_{uw}	5,19	5,32,20	5,32,20,21	5,32,22	5,23	5,24	5,25	5,26	5,32,27	5,28	5,32,29	5,32,29,30	5,32,31	5,32	5,33	34	35	36
H_{un}	6,19	6,20	6,21	6,32,22	6,23	6,7,24	6,32,25	6,20,26	6,32,27	6,28	6,29	6,30	6,32,31	6,32	6,32,33	34	35	36
H_{ue}	7,19	7,20	7,20,21	7,22	7,23	7,24	7,32,25	7,26	7,32,27	7,28	7,29	7,20,30	7,31	7,32	7,32,33	34	35	36
H_{rk}	8,19	8,20	8,20,21	8,32,22	8,23	8,7,24	8,32,25	8,20,26	8,32,27	8,28	8,29	8,29,30	8,32,31	8,32	8,32,33	34	35	36
H_{rs}	9,19	9,20	9,20,21	9,32,22	9,23	9,7,24	9,32,25	9,20,26	9,32,27	9,28	9,29	9,29,30	9,32,31	9,32	9,32,33	34	35	36
H_{rw}	10,19	10,20	10,20,21	10,32,22	10,23	10,7,24	10,32,25	10,20,26	10,32,27	10,28	10,29	10,29,30	10,32,31	10,32	10,32,33	34	35	36
H_{rn}	11,19	11,20	11,20,21	11,32,22	11,23	11,7,24	11,32,25	11,20,26	11,32,27	11,28	11,29	11,29,30	11,32,31	11,32	11,32,33	34	35	36
H_{re}	12,19	12,20	12,20,21	12,32,22	12,23	12,7,24	12,32,25	12,20,26	12,32,27	12,28	12,29	12,29,30	12,32,31	12,32	12,32,33	34	35	36

H_{tra}	13,19	13,20	13,20,21	13,32,22	13,23	13,32,7,24	13,32,25	13,20,26	13,32,27	13,28	13,29	13,29,30	13,32,31	13,32	13,32,33	34	35	36
F_i	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
P_a	15,19	15,20	15,20,21	15,32,22	15,23	15,20,7,24	15,25	15,20,26	15,27	15,28	15,29	15,29,30	15,32,31	15,32	15,33	34	35	36
P_{ax}	16,32,13,19	16,20,17,20	16,20,21	16,22	16,23	16,20,7,24	16,25	16,20,26	16,27	16,32,13,28	16,29	16,29,30	16,31	16,32	16,33	34	35	36
P_{fm}	17,19	17,20	17,20,21	17,22	17,23	17,20,7,24	17,25	17,20,26	17,27	17,28	17,29	17,29,30	17,31	17,32	17,33	34	35	36
P_m	18,13,19	18,20	18,21	18,22	18,23	18,7,24	18,32,25	18,20,26	18,32,27	18,13,28	18,29	18,30	18,31	18,32	18,32,33	34	35	36
P_s	19	19,20	19,21	19,22	19,23	19,7,24	19,25	19,20,26	19,32,27	19,13,28	19,29	19,30	19,31	19,32	19,33	34	35	36
P_{pudm}	20,13,19	20	20,21	20,22	20,23	20,7,24	20,32,25	20,26	20,32,17	20,13,28	20,29	20,30	20,31	20,32	20,32,33	34	35	36
P_e	21,13,19	21,20	21	21,22	21,23	21,7,24	21,25	21,20,26	21,27	21,13,28	21,29	21,29,30	21,31	21,32	21,32,33	34	35	36
P_{he}	22,113,19	22,20	22,20,21	22	22,23	22,7,24	22,25	22,20,26	22,32,27	22,13,28	22,29	22,29,30	22,31	22,32	22,33	34	35	36
P_{fp}	23,13,19	23,20	23,20,21	23,22	23	23,7,24	23,25	23,20,26	23,32,27	23,13,28	23,29	23,29,30	23,31	23,32	23,33	34	35	36
C_a	24,19	24,20	24,20,21	24,32,22	24,23	24	24,25	24,20,26	24,27	24,28	24,29	24,29,30	24,32,31	24,32	24,32,33	34	35	36
C_{ax}	25,32,13,19	25,20	25,20,21	25,22	25,23	25,20,7,24	25	25,20,26	25,27	25,32,13,28	25,29	25,29,30	25,31	25,32	25,32,33	34	35	36
C_{fm}	26,32,13,19	26,20	26,20,21	26,22	26,23	26,7,24	26,25	26	26,27	26,32,13,28	26,29	26,29,30	26,31	26,32	26,32,33	34	35	36
C_m	27,19	27,20	27,21	27,22	27,23	27,24	27,32,25	27,26	27	27,28	27,29	27,30	27,31	27,32	27,32,33	34	35	36
C_s	28,13,19	28,20	28,21	28,22	28,23	28,24	28,25	28,26	28,27	28	28,29	28,30	28,31	28,32	28,33	34	35	36
C_{pudm}	29,13,19	29,32,20	29,21	29,22	29,23	29,7,24	29,32,25	29,26	29,32,27	29,13,28	29	29,30	29,31	29,32	29,32,33	34	35	36
C_e	30,13,19	30,20	30,20,21	30,22	30,23	30,7,24	30,25	30,20,26	30,27	30,13,28	30,29	30	30,31	30,32	30,32,33	34	35	36
C_{he}	31,13,19	31,20	31,20,21	31,32,22	31,23	31,7,24	31,25	31,20,26	31,27	31,18,28	31,29	31,29,30	31	31,32	31,33	34	35	36
C_{fp}	32,13,19	32,20	32,20,21	32,22	32,20,23	32,7,24	32,25	32,20,26	32,27	32,13,28	32,29	32,29,30	32,31	32	32,33	34	35	36
C_{tm}	33,13,19	33,20	33,21	33,22	33,23	33,24	33,25	33,26	33,27	33,13,28	33,29	33,30	33,31	33,32	33	34	35	36
X_a	34,19	34,20	34,20,21	34,32,22	34,23	34,20,7,24	34,25	34,20,26	34,27	34,28	34,29	34,29,30	34,32,31	34,32,	34,33	34	35	36
X_{mi}	35,19	35,20	35,20,21	35,22	35,23	35,20,7,24	35,25	35,20,26	35,27	35,28	35,29	35,29,30	35,31	35,32	35,33	34	35	36
X_m	36,13,19	36,20	36,21	36,22	36,23	36,7,24	36,32,25	36,20,26	36,32,27	36,13,28	36,29	36,30	36,31	36,32	36,33	34	35	36

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